VOLUME LVI NUMBER 8 WHOLE 496

SCHOOL SCIENCE AND MATHEMATICS

NOVEMBER 1956

School Science and Mathematics

A Journal for All Science and Mathematics Teachers

All matter for publication, including books for review, should be addressed to the editor. Payments and all matter relating to subscriptions, change of address, etc. should be sent to the business manager.

Entered as second class matter December 8, 1932, at Menasha, Wisconsin, under the Act of March 3, 1879. Published Monthly except July, August and September at 450 Ahnaip St., Menasha, Wis. PRICE. Four dollars and fifty cents a year; foreign countries $5.00; current single copies 75 cents.

Contents of previous issues may be found in the Educational Index to Periodicals.

Printed in the United States of America

When God afflicts thee,
Think He hews a rugged stone,
Which must be shaped, or else
Aside as useless thrown.—FRENCH.

CONTENTS FOR NOVEMBER, 1956

CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS, INC.

APPLICATION FOR MEMBERSHIP

Date ....19.........

I hereby apply for membership in the Central Association of Science and Mathematics Teachers, Inc., and inclose $3.50 for annual membership dues, which includes one year's subscription to SCHOOL SCIENCE AND MATHEMATICS. I will receive nine issues of this journal, none being published in July, August or September. (Subscriptions beginning with January issue expire with December; with October issue expire with June.)

Begin: JANUARY ☐ Begin: OCTOBER ☐

(PLEASE PRINT)

Name ....
Last Name First Name

School Address ....
Name of School City State

Home Address ....
Street City Postal Zone State

Journals will be sent to home address unless otherwise requested

Underline Section in which enrollment is desired: *Biology, Chemistry, Elementary Science, Elementary Mathematics, General Science, Geography, Mathematics, Physics* ....

Mail this application with $3.50 (Canada $3.75, Foreign $4.00) to Central Association of Science and Mathematics Teachers, Inc., P.O. Box 408, Oak Park, Ill.

PLEASE CHECK IF YOU FORMERLY WERE A MEMBER ☐

CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS

OFFICERS FOR 1956

President: Charlotte W. Junge, Wayne University, Detroit, Mich.
Vice President: James H. Otto, Washington High School, Indianapolis, Ind.
Secretary-Historian: Virginia Terhune, Proviso Township High School, Maywood, Ill.
Treasurer-Business Manager: Ray C. Soliday, Oak Park High School, Oak Park, Ill.

EXECUTIVE COMMITTEE FOR 1956

Charlotte W. Junge, President
James H. Otto, Vice President
Milton O. Pella, President for 1955

BOARD OF DIRECTORS FOR 1956

Ex-Officio

Charlotte W. Junge, President
James H. Otto, Vice President
Milton O. Pella, President, 1955, University of Wisconsin, Madison, Wis.

Terms Expire in 1956

William Herbert Edwards, Central High School, Detroit, Mich.
Jacqueline Buck Mallinson, 535 Kendall Avenue, Kalamazoo, Mich.
Louise Panush, Northeastern High School, Detroit, Mich.
Reino Takala, Hinsdale High School, Hinsdale, Ill.

Terms Expire in 1957

E. Wayne Gross, Laboratory School, Indiana University, Bloomington, Ind.
Ralph C. Huffer, Beloit College, Beloit, Wis.
Clyde T. McCormick, Illinois State Normal University, Normal, Ill.
Willard D. Unsicker, San Diego, Calif.

Terms Expire in 1958

Edward Bos, Proviso Township High School, Maywood, Ill.
Sister Mary Evarista, Mercy High School, Chicago, Ill.
Cecil Read, University of Wichita, Wichita, Kan.
F. Lynwood Wren, George Peabody College, Nashville, Tenn.

EDITOR OF THE YEARBOOK

Luther Shetler, Bluffton College, Bluffton, Ohio

JOURNAL OFFICERS

Glen W. Warner, Editor, Lakeville, Ind.
George G. Mallinson, Assistant Editor, Western Michigan College of Education, Kalamazoo, Mich.

SECTION OFFICERS FOR 1956

(Order: Chairman, Vice Chairman, Secretary)

Biology

Robert A. Bullington, Northern Illinois State Teachers College, DeKalb, Ill.
Lulu M. Casemore, Mackenzie High School, Detroit, Mich.
William Hill, Naperville High School, Naperville, Ill.

Chemistry

Brother Fred T. Weisbruch, Don Bosco High School, Milwaukee, Wis.
Robert L. Walker, Lyons Township High School and Junior College, LaGrange, Ill.
Arthur Reynolds, Northern High School, Detroit, Mich.

Conservation

Richard Weaver, University of Michigan, Ann Arbor, Mich.
Marvin R. Bell, Indiana State Teachers College, Terre Haute, Ind.
Iva Spangler, Central High School, Fort Wayne, Ind.

Elementary Mathematics

Mrs. Clarence Ethel Hardgrove, Northern Illinois State Teachers College, DeKalb, Ill.
Elinor B. Flagg, Illinois State Normal University, Normal, Ill.
Sheldon Myers, Educational Testing Service, Princeton, N. J.

Elementary Science

Mrs. Emilie Lepthien, Board of Education, Chicago, Ill.
Frank O. Youkstetter, Wayne University, Detroit, Mich.
Sister Mary Lorita, St. Xavier College, Chicago, Ill.

General Science

Phil R. Gilman, East High School, Cleveland, Ohio
Fred L. Keil, Proviso Township High School, Maywood, Ill.
Sister Mary Ambrosia, Immaculata High School, Detroit, Mich.

Mathematics

Luther Shetler, Bluffton College, Bluffton, Ohio
Hobert Sistler, J. Sterling Morton High School, Cicero, Ill.
Leonard Antel, Cass Adult Day School, Detroit, Mich.

Physics

Edwin Vaughn, Western Michigan College Paw Paw, Mich.
Robert L. Price, Joliet Junior College, Joliet, Ill.
Robert Eugene Carpenter, Richmond Senior High School, Richmond, Ind.

SCHOOL SCIENCE AND MATHEMATICS

VOL. LVI | NOVEMBER, 1956 | WHOLE NO. 496

THE CRITICAL SHORTAGE OF SCIENCE AND MATHEMATICS IN OUR SCHOOLS TODAY

J. W. GALBREATH

East St. Louis High School, East St. Louis, Illinois

IS THERE A PROBLEM?

In 1954 American Colleges turned out 57% fewer High School Science Teachers than in 1950 and 51% fewer Mathematics Teachers than in 1950.

One half the high schools in the United States do not offer a course in physics or chemistry.

Since 1900 students studying algebra have fallen from 56% to 24%. Chemistry has dropped from 27% to 11%.

In 1954 the percentage of high school students who take physics is 4.3%; in 1900 it was 19%.

Recently numerous articles have appeared in leading newspapers stressing the critical shortage of scientists, mathematicians and engineers.

According to studies made by Admiral Strauss and Admiral Richover "this shortage is more dangerous to the United States than the Red Army's 300 odd divisions, Moscow's 400 or more submarines, its present stock piles of A-bombs and H-bombs; the Kremlins propaganda campaign; her competitive co-existence policy, or her anticolonization drive."

This takes in a lot of territory but the stituation according to Admiral Strauss is in danger of robbing us of "All of our blessings, even our freedom" and "Our present situation is dangerous; in a decade it could be disastrous."

Says Admiral Richover "This is serious and we have failed, and it

is the fault of all of us, the way we meet our future challenge will determine the future of our country."

"The United States is training fewer than half of the scientists and engineers needed at the present and the future looks even worse." The function of education has been stated to educate all children up to the capacity of their ability to think clearly and act impartially. The question arises, are we in our American school systems properly developing the upper per cent of better students to their fullest capacity to benefit?

How Are the Russians Doing?

"The Soviets are training twice as many engineers and scientists as we are. Here we are training 500 Nuclear scientists when we need 2000. The real bottle neck here is the need of scientists to teach these scientists. The Soviets have a longer school year, longer school week and shorter vacation. They work the students harder in school hours. They seek out the gifted students and give them every encouragement to get ahead. They stress physics, chemistry and mathematics. They know the value of scientists, engineers and physicists and are pushing their training. The United States has no real program to select, encourage, and train these needed specialists upon which our future depends."

Dr. W. V. Lambert, dean of the University of Nebraska, College of Agriculture, who headed the American farm delegation to Russia last summer, stated recently, "Whether we like it or not we can't laugh off Russia's tremendous progress in industry, agriculture, education, science and research. The Soviet Union is turning out 50,000 College trained engineers annually as compared with some 17,000 in the U. S.; 20,000 graduates of medical schools, almost three times the 7500 here; and two and one-half times the number of agricultural college graduates. The same is true in the field of physics, chemistry, and mathematics. They are way ahead of us. At the present rate in another 25 years or so their technological man power pool will be so far ahead of ours that we simply won't be able to compete with them for world leadership."

Why Don't Students Choose Science and Mathematics Courses Today?

We live in a scientific age, but like the proverbial ostrich we bury our heads in the sands of indifference and refuse to become concerned. We push buttons and turn keys and marvels are performed but as to *how* and *why* we are too indifferent or too lazy mentally to find out or understand the principles involved.

Why don't boys and girls of ability in our schools today take the standard science and mathematic courses? They are naturally curious about their environment. An average six year old will ask as many questions today as his grandfather would ask at the same age in 1900. Do we dull this (want to know attitude) as they go through our schools? Do we challenge them sufficiently to achieve their best? Does the good student learn to enjoy loafing through school rather than accomplishing tangible results? Have our best students failed to appreciate and enjoy accomplishment? Have we made education unpopular for the better students by watering it down to where it is popular with the masses?

Is it wrong to take a course because it requires some real effort to master the essentials necessary to do reflective thinking? Mary doesn't take biology because Jane is not taking it. Jane talked to Susan, who had taken the course, and she reported it required too much work. Bill, a capable student, would rather shine on the basketball floor than he would in the physics laboratory. Have we lost our sense of values as to what is good, permanent, and has a future? Have too many of us chosen or fallen into the easy way, losing the fruits and satisfaction of toil and sweat?

Have scientists made life so easy for the majority of us that we no longer need or care to attempt to understand it? Have we failed to challenge the bright students by offering watered down, sugar coated. hybrid courses that they take for an "easy credit," without exerting themselves or ever tasting the pleasure of real accomplishment?

Boys and girls, in my opinion, are the same today as they were in 1900, as far as their abilities, capacities, and desires are concerned. They can do the "tough" courses but they must have the proper guidance and incentives. I believe we have failed to sell science and mathematics in the elementary and junior high schools.

Students need inspiration through "pep talks" exalting the virtues and need of science and mathematics. They need to know the requirements of medicine, dentistry, nursing, teaching, laboratory technician, engineering, nuclear physicst, and industrial chemist. They need to know of the opportunities in science, the opportunities for serving their fellow men, or their country to the best of their God-given abilities and capacities. They need to know about the many sources of assistance, awards, scholarships, fellowships, Science Talent Search, and Science Fairs.

We need to have teachers with at least enough scientific background to stimulate, lead and sympathize with the budding scientists in their classrooms, even in the 4th, 5th, and 6th grades. If students elect careers in science and mathematics in college, it is generally the result of the background, appreciation of and inspiration for science,

they received from their teachers in the elementary, junior high school and high school.

There is a greater need for a plan of action (as to what to do now) than there is to rationalize as to why existing conditions are what they are today.

What Can Be Done?

1. We can publicize what our science and mathematics courses are. We have put out untold propaganda on the virtues of how "white shine tooth paste" makes your teeth clean and white but forget to inform the user that it is also necessary to eat the right kind of foods to have good teeth. We have often reveled in the marvels of what science does and neglected to know what science is. Many teachers do not know that an engineer needs calculus before he can be efficient at his work; they also neglect to tell their students that they must have elementary algebra and plane geometry as a pre-requisite to trigonometry and calculus.

Through proper counseling and guidance the youngster can be shown the need for biology, physics, and chemistry in being a healthy and happy citizen in a complex scientific world.

The teacher needs to create a respect for enjoyment in accomplishment and knowledge for its own sake. Too many educators and students have been led to believe that Johnny should never do anything distasteful, difficult or stimulating in fear of developing an inferiority complex or experience frustration; it could be that we need to get back to the fundamentals that mental exertion is not only beneficial but exhilarating. Many have experienced intense pleasure in intellectual accomplishment. Many have expressed a thrill and joy in having successfully solved a difficult problem. The pupil needs to experience the thrill and award of a job well done. The teacher needs to surround the pupil with an atmosphere of learning experiments with simple equipment. The elementary curriculum must provide time and materials for the science period. Too often it comes at the end of the day and is skipped.

The educational system must provide each classroom with a teacher, who has had some background in science.

Courses in science must be adapted to the background of the student and give him an opportunity to participate. They should be organized on the psychological rather than the logical order. They should stress the social values of living as well as the scientific. A wider and more effective use of audio-visual aids should be used; no subjects lend themselves to as broad treatment audio-visually as do the sciences. It is no longer the case that the question arises *is there*

a film we can use? But what is the *best* film to use? We can use booklets, pamphlets and films to sell the student on the need for science and mathematics. We can visit industry and invite outside specialists into our schools to help us put across the need for more science and mathematics. The pupil must know the way before he arrives at the tenth grade, if he is going to do much about it. It is too late for the student to plan a high school curriculum in the Junior or Senior year in high school.

We can provide more time for science in our high schools. Most high schools operating on the hour period have eliminated the double laboratory period. Still when the teacher takes science in college he is required to spend two or three times as many hours in the laboratory as he does in taking courses in education, English, or social study classes. His pay is no higher in the average school system than any other teacher, because most school systems salary schedules are operated on the ideal of "equal pay for equal work," "a degree is a degree."

Still physics and chemistry are being dropped by the smaller schools and are on their way out unless a definite change in trend is brought about and that very soon.

What Some Schools Are Doing

A. Selecting and segregating their gifted students with science potential by the use of specific aids or criteria such as:

1. Genetic ability by general intelligence tests. I. Q. basis.

2. Predisposing Factors—a tendency or leaning toward science, a certain questing after and persistence in scientific endeavor. Many of these future scientists will identify themselves if given the opportunity to take advantage of enriched programs. They want to become scientists.

3. Use of activating incentives which provide: (a) the environmental opportunities for scientific expression and satisfaction. (b) Leadership, guidance, and skills of a sympathetic teacher.

B. According to the study of Paul Brandwine in his recent book "The Gifted Student as Future Scientist," there are certain guiding principles that bring results, they include.

1. There are known methods by which gifted students can be selected for scientific careers. These boys and girls need to and desire to become scientists. In the past we have too often operated on the theory that these gifted students could take care of themselves in the regular classroom. This theory has resulted in a great waste of human resources.

2. Those gifted students can be trained to become scientists and

mathematicians if proper opportunities for their development are provided and if they are properly activated.

3. Our nation needs scientists. Our schools can supply this need. There is a method that works. The future of science depends upon the science program in operation in our schools today.

4. Many students of high ability who would profit by a college education, for one reason or another never get to college. Many of these could be trained to become capable career men in science and related fields. These wasted resources must be identified early in their school careers and proper means provided for their taking college training.

5. Providing an enriched opportunity in science to properly activate those selected, including:

a. Environmental opportunities—special laboratory space and equipment made available.
b. Selecting and completing research projects for class use and exhibits, science talent search contests, science fairs, future scientists of America, etc., etc.
c. Making ably trained laboratory assistants as aids in training other students in small groups of four or five.
d. Enriching opportunity for reading in science such as Scientific American, Science News Letter, Natural History, and latest reference books including some on college level in science.
e. Personal interviews with the teacher, man to man ratings. Inspiration, appreciation, encouragement, counsel, and guidance. The essential activating incentive to more scientists in the future is the teacher.
f. Providing advance courses in special sciences.

Reference on Gifted Students as Future Scientists

1. "Five Basic Ways to Improve Science Courses." Mark-Kent State University, School Science and Mathematics, December, 1955.
2. "Survey Research in Elementary School." *Science Education*, Buck and Mallinson at Western Michigan College of Education, School Science and Mathematics, December, 1955.
3. "Illinois Education." *Science Survey*. By Shell Oil Co. Foundation, Decmber, 1955.
5. "Uncle Sam's Achilles' Heel." Neal Stanford, *Christian Science Monitor*, 1955.
6. "Science Teaching Improvement Program." 1955 A.A.A.S., Washington, 5, D. C.
7. "Encouraging Future Scientists of America Foundation," N.S.T.A.
8. "The Rabbit with the High IQ." Bureau of Publications, Teachers College, Columbia University, N. Y., 1955.
9. "The Gifted Child in the Regular Classroom." Bureau of Publications, Teachers College, Columbia University, N. Y., 1955.
10. "The Role of the Secondary School in the Teaching of Science." *Physics Today*, Vol. 5, No. 6, June, 1952.

11. "Raising the Masses Is a Problem in Physics." Cope, SCHOOL SCIENCE AND MATH., December, 1942.
12. "Your Opportunities in Science and Engineering." National Association of Manufacturers. N. Y. 17, N. Y., 1955.
13. "Science Club Manual and Projects." Science Clubs of America, Washington D. C., 1955.
14. "Science Achievement Awards for Students and Recognition Awards for Science Teachers." N.S.T.A., Washington, D.C.
15. "I Am the Science Teacher." Irving, *American School Board Journal*, May, 1955.
16. "Science Teaching at the Crossroads." Van Fleet, N. J. Science Teachers Association, Rutgers University, May, 1954.
17. "Teaching Conditions and the Work Week of High School Science Teachers." N.S.T.A., Washington, D.C., 1948.
18. "The Teaching of Biology in Secondary Schools of the U. S." Riddle and Others, Union of America. Biology Societies Carnegie Institute of Washington, D. C., 1942.
19. "The Teaching of Science in Public High Schools." Bulletin No. 9, Federal Security Agency, Office of Education, Washington, D. C., 1950.
20. "The Gifted Child." Paul Wittey, The American Association for Gifted Children, Heath, 1951.
21. "Is Home Work One Answer?" Worlever, SCHOOL SCIENCE AND MATHEMATICS, December, 1955.
22. "Young Scientists Studied." *Science News Letter*, January 21, 1956.
23. "Science Teaching Ideas." N.S.T.A., Washington, D. C. 1955.
24. "The National Science Foundation Program." Kelson. Also "Problems of Teaching Biology." North Central Conference on Biology Teaching, N.A.B.T., *The American Biology Teacher*, January, 1956.
25. "The Superior Pupil in Junior H. S. Mathematics." McWilliams and Brown, Office of Education. Bulletin No. 4, Supt. of Documents, U. S. Government Printing Office, Washington, D. C. 1955. 25¢.
26. "What Are You Teaching?" Keller, SCHOOL SCIENCE AND MATHEMATICS, March, 1956.
27. "The Growing Emphasis on Science and Mathematics in Industry." Luft, SCHOOL SCIENCE AND MATHEMATICS, March,1956.
28. "The Gifted Student as Future Scientist." Paul Brandwein. Harcourt, Brace & Co., 1955.
29. "Some Implications of Problem Applications of Recent Research in Science Education. No. 2, Mallinson and Buck, SCHOOL SCIENCE AND MATHEMATICS, May, 1956.

EUROPEAN STUDY TOUR

Wayne State University's College of Education and Graduate School again approve credit arrangements in connection with the Tenth Annual ***European Study Tour in Comparative Education***. Personally conducted by Dr. Wm. Reitz, Professor of Education, the tour will leave Detroit on June 21, 1957, and return on August 24, 1957.

Visiting 10 countries in 9 weeks, this tour is designed to provide teachers, students, and professional people with an opportunity to survey selected highlights of the life and culture of Western Europe.

Qualified persons may earn up to 8 hours of undergraduate or graduate credit to apply on degree programs, for teaching certification, for annual salary increments, or for personal enrichment.

Further information may be obtained from Dr. Wm. Reitz, 727 Student Center, Wayne State University, Detroit 2, Michigan.

SOME THOUGHTS ON GENERAL SCIENCE AND GENERAL SCIENCE TEACHER PREPARATION[1]

RICHARD R. ARMACOST
Purdue University, Lafayette, Ind.

General science should be a key program in every school. Early secondary school students are usually enthusiastic and ambitious. They are full of uncorralled energy. Since general science may be the only science taken by most of them, it should fulfill certain requisites in their education.

For example, the general science program should offer a general overview of the different science areas, including health and safety, with a strong emphasis on topics of local interest and importance. Applications of the material studied should be numerous. Students should have the opportunity to become acquainted with simple techniques and have the fun of finding answers to their questions by experimentation. They should take field trips and go on excursions to local industry. They should be encouraged to participate in science fairs and congresses. Because these and other interesting activities are frequently omitted from the general science program, most pupils leave the ninth grade cold to science and science teachers. The superficiality of the science program and particularly the lack of action in it has given them little or no incentive to continue further study, either formally or informally.

A teacher especially prepared with a broad science background and equipped with efficient teaching procedures is needed to meet the requirements for the development of good general science programs, and good general science teaching. Many diverse ideas exist concerning the training of general science teachers. There are those who still think that "anyone" can teach general science. Some think training should be little different than that of the more specialized biology, chemistry, or physics teacher. Others point to the general science teacher as a unique person who needs a very specific type of preparation. Science course work offered by colleges and universities varies in organization, but the following program or its equivalent represents a good basic training for a prospective general science teacher.

Laboratory course work is needed in physics, chemistry, botany, zoology, geology, microbiology, and descriptive astronomy. Universities and colleges should study ways of presenting needed instruction in as few courses as possible. This means that some traditional courses will be consolidated into new integrated courses. In addition to elementary study, the prospective general science teacher should

[1] Presented to the Workshop on "Organizing Subject Matter for Junior High School Teachers," June 14–18, 1955, Oswego, New York.

have advanced work in one, or preferably two, of the areas listed above.

In professional education training, emphasis should be on: adolescent growth and development with adolescent group experience; a study of present day types of school systems, including major objectives of the total program and how they can be improved; modern science methods and practice in using them; evaluation and a study of the total role of the teacher including the relationship of the community and school with practice teaching and living in a "typical" community.

Throughout this program of science and professional education certain procedures should be stressed. The importance of learning by first-hand experiences needs to be recognized continually. Individual pupil laboratory work, field trips, museum trips, excursions, project work, and "asking someone who knows" should become primary steps in the prospective science teacher's plans for teaching children science. He should not fail to learn the techniques involved in laboratory work, test construction, including modern types of test items, direction of field studies, guidance of students in project work, and curriculum building. Since many learning difficulties of his students can be traced to reading deficiencies, he needs to appreciate the importance of reading in learning science.

With these problems and suggestions in mind, the main sources of present day general science teachers need some description. Few have been particularly trained for general science jobs. Many are serving an apprenticeship in general science before "advancing" to the senior high school. Some are the best elementary school teachers who have been "promoted" to the junior high school. As one teacher so aptly has put it, "Many teachers of general science are borrowed from other departments to fill a general science emergency which never ends."

The seriousness of this situation is magnified when it is realized that the best of teachers may have two administrative strikes against him in many of the present day general science programs. Classes are too large and rooms too small; too little time is allotted; teachers are required to teach the same lesson over and over on a given day until they are just plain bored; teacher load is too great; little or no money is allocated for books and simple equipment; so-called experts, as well as colleagues in the senior sciences, give little or no help and seem to care less for the plight of the general science teacher.

High school administrators frequently criticize colleges and universities for lacking the comprehensive programs necessary for production of well-trained general science teachers. One able principal of a large centralized school recently asked, "How can I find a good

general science teacher when all the applicants for a science job are specialists in physics, chemistry, or biology?"

It is true that many colleges and universities have not developed satisfactory programs for training general science teachers. However, some colleges and universities have had this type of education for a number of years. Placement bureau heads claim that high school administrators seldom ask for a general science teacher, but want someone who can teach something else *and, if need be*, general science.

Although it is difficult to teach general science well, it is fairly easy for mediocre and even poor teaching in this area to escape criticism from high school administrators, parents and students. Too many teachers of general science are not capable general science teachers. Most high school administrators never realize the importance of general science and good general science teaching. There is more or less a universal feeling that students are "not expected to learn much" in the general science program. *Consequently, they don't.* General science is not usually a very successful part of the early secondary school curriculum.

What should be done? First, the importance of the general science program must be more fully appreciated by state departments of education, high school administrators, trustees, school boards, teachers, and parents. In particular, those who hire new teachers need to recognize the necessity for competent general science training. Second, teachers in-service who lack adequate training should be encouraged continually to remedy their deficiencies by college and university attendance, travel and other types of education, or by various types of in-service teacher education. Third, colleges and universities need to train more teachers primarily for general science teaching, and in addition should provide better and more in-service teacher education, as well as counseling services.

RESEARCH SHIP TO VIEW SEA-AIR BOUNDARY

A new scientific look at the sea and the air above it will be the mission of the New York University research schooner *Action* being commissioned (Aug. 22) for science.

This 65-foot two-masted craft will first be used to measure the exchange of energy between the atmosphere and the ocean's surface. Interaction at the air-sea boundary is important in weather forecasting.

Tiny ocean waves will be investigated to determine how they reflect radar signals and reduce radar's usefulness in navigation.

The oceanographic schooner *Action* will later study tides and currents in the New York area, relating the findings to water pollution, bathing areas, shellfish and erosion of coastal areas.

The *Action* has a wide cruising radius and its construction makes it exceptionally stable while observations are being made.

GEORGE EDMON HAWKINS
1901–1956

Another member of our staff has quietly passed to the Great Unknown. It is almost twelve years since George accepted the editorship of our high school mathematics department. In this time he has served the *Journal* well as a critic of the articles placed in his hands for examination and judgment, as a book reviewer, and consultant on journal affairs. For many years he has contributed his time and excellent judgment to the Central Association of Science and Mathematics Teachers serving as a member of the Board of Directors, a section officer, Vice-president in 1944–45, and President in 1945–46. As a teacher and school executive Lyons Township High Schools and Junior College found him one of the great. The following sentences from his fellow teachers tell of his manner and quiet greatness as a teacher and friend:

> "The soft-spoken way in which he greeted us in the halls, the friendly manner with which he welcomed us into his office, his interest in student activities whether within or outside the classroom, the gentle manner in which he handled the problems of a large school, his thoughtfulness, his kindness, his patience, his sincerity in all matters—these are the things we shall remember. And we shall remember, too, his high standards, his integrity, and his concern for the smooth running of the school. So dedicated was he to that purpose that even during the last hours of his life, while desperately ill, he was directing others in the planning of the approaching school year. This selfless act was characteristic of his devotion to duty."

He was born at Noble, Illinois, October 4, 1901. After graduation from high school he attended the Eastern Illinois State Teacher's College at Charleston in 1922–23. His teaching work began in the rural schools of Richland County, Illinois, in 1920–22. Next in grades 9 and 10 at Jewett, 1923–25, principal and mathematics teacher at Lake Zurich, 1925–27, and at Highland Park, 1927–30. His next move was to The University of Chicago High School after receiving his B.S. Degree from Chicago in 1930. Here he taught and continued his education in mathematics and in the School of Education, receiving the Master's Degree in 1934, and continuing work toward the Doctors Degree until 1941. He then transferred to La Grange as head of the mathematics department of Lyons Township High School and Junior College.

He with Gladys Tate, was the author of *Your Mathematics* published by Scott Foresman and Company, and used in schools throughout the country.

His death occurred suddenly of a heart attack at McNeal Memorial Hospital, Berwyn, Illinois, August 15. His wife Bertha, a son, Robert A. and a daughter, Mrs. Ruth Ann Vein, are the members of

his immediate family. Burial was at Olney, Illinois, his old home town.

Prepared by GLEN W. WARNER
with the assistance of
FRANK B. ALLEN and
Others of Lyons School

IN DEFENSE OF THE "SIX" GREAT LAKES

JAMES K. ANTHONY
Southern University, Baton Rouge, Louisiana

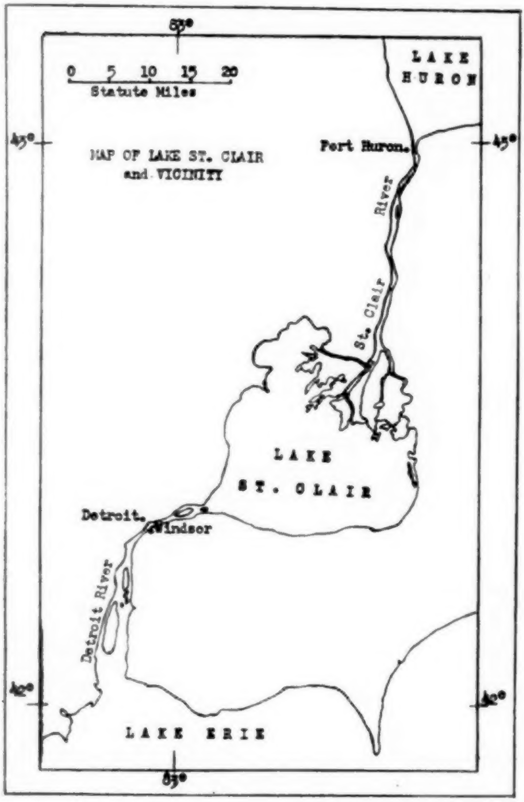
If the old adage that *a chain is as strong as its weakest link* is true then the principle would hold true in the case of our Great Lakes of North America. Many geographers and a multitude of textbooks refuse to recognize the fact there are *six* rather than five lakes that are commonly known as the Five Great Lakes or simply The Great Lakes. Ever since the retreat of the Wisconsian Ice Sheet during the late Pleistocene period about 12,000 years ago, which left the topography of the northern half of North America essentially the same as it is today, the Great Lakes have been a conspicuous feature of the landscape.

If the question of size is the lone criterion then one can logically and quite truthfully say there are five Great Lakes namely: Lake Superior, 31,810 square miles; Lake Huron, 23,010 square miles; Lake Michigan, 22,400 square miles; Lake Erie, 9,940 square miles; and Lake Ontario, 7,540 square miles. But if one of the lifelines of the nation is involved to the extent that our economic existence is threatened then one must re-appraise his standards or boundaries of definitions. If size is the basis for determining the limits of a lake, the diminutive Lake St. Clair would not be thought of as belonging to this system of waterways. For it is only about 460 square miles and has to be dredged continually to boast of a 19-foot depth. Moreover, navigation from the upper lakes to the lower lakes would be impossible without Lake St. Clair's important location.

Situated between the state of Michigan and the province of Ontario, Lake St. Clair receives water from Lake Huron by way of the St. Clair River and flows into Lake Erie through its southern outlet, the Detroit River (See map). The fundamental importance of Lake St. Clair lies in its aid to navigation. Imagine an attempt to travel *via* boat from Duluth, Minnesota to Cleveland, Ohio if Lake St. Clair did not exist and in its place was a land bridge connecting the United States with Canada. The trip would be impossible. In

fact, the importance of Lake St. Clair is probably greater than that of the Soo Canal.

In 1800 the efforts of the United States to venture into the steel industry had become firmly established in the American economy and the skies of the lower Great lakes region were illuminated nightly in multicolors from the fires of the busy blast furnaces. By 1850 one half-million tons of iron ore had been shipped from Minnesota's mesabi mines. The Soo Canal was opened in 1855 to facilitate the northbound ships that encountered tremendous difficulties navigating



the rapids of the St. Mary's River. And by 1860 over one million tons of precious hematite had found its way to the hungry iron and steel mills of Ohio and Pennsylvania.

Imagine the increased cost of freight if there existed no Lake St. Clair and the iron ore had to be transshipped overland by railroads with their notoriously high freight rates? How different would the

arguments for the proposed Great Lakes-St. Lawrence Seaway have been if there were no Lake St. Clair? It seems dubious that the Maple-Leaf government would have been anxious to tackle the seaway project if constructing a passageway from Lake Huron to Lake Erie had to be considered, too.

Other industries on the American scene have flourished because of the unbroken chain of lakes, e.g., the transportation of wheat from the glacially-rich Lake Agassiz region to the eastern seaboard and Europe; the limestone-mining activities of the Great Lakes region—the necessary partner in the steel industry; the northward shipment of coal from the United States to Canada; the shipping of automobiles from Detroit to other Great Lakes cities; and the fast-declining tourist trade for which these lakes were internationally famous to cite a few examples.

Thus based on man's economic activities, and they are essential if he is to continue to enjoy this chromium-plated, breath-taking civilization he has created, then let us take cognizance of all of the factors and facets that have made this existence possible. Or stating the question in other words: how long can educators justify their *not* recognizing the importance and strategic location of Lake St. Clair?

SCHOOL AND COLLEGE ENROLLMENTS

The Nation's total school and college enrollment will reach an all-time peak of 41,553,000 in 1956–57, S. M. Brownell, Commissioner of Education, U. S. Department of Health, Education, and Welfare, estimated in a report released last month.

Reporting on other trends, Commissioner Brownell said shortages of classrooms and qualified teachers will be somewhat reduced, although still serious.

"This will be the twelfth consecutive year that the total enrollment of schools, colleges, and universities has shown an increase," Commissioner Brownell said. "The 1956–57 estimated enrollment will be 1,754,300 higher than the previous peak enrollment of 39,798,700 recorded in 1955–56."

Enrollment estimates released by the Office of Education show that private and public school enrollment in Kindergarten through Grade 8 will total 29,618,000. Last year's elementary school enrollment was 28,514,200. The increase this year is 1,103,800.

High school enrollment for 1956–57 is estimated at 8,111,600. This is a step up of 364,500 high school students (Grades 9 through 12) over the 1955–56 total enrollment of 7,747,100.

Colleges and universities throughout the United States will enroll 236,000 more students during the coming academic year than they enrolled in 1955–56. This year's estimated enrollment will be 3,232,000 as compared with last year's 2,996,000.

The shortage of 120,700 qualified elementary and secondary teachers will have to be met by additional emergency teachers, by the re-entrance of former teachers into the profession, and by further overcrowding. In the calculations of this figure, no provision was made for additional teachers to reduce present overcrowding or to enrich the curriculum.

A STUDY OF THE RELATIVE EFFECTIVENESS OF GROUP INSTRUCTION

DONOVAN A. JOHNSON
University of Minnesota High School, Minneapolis, Minn.

One of the most commonly advocated methods of providing for individual differences within a class is to divide the class into groups. These groups are then given instruction and proceed independently of each other. This group method has been suggested for secondary mathematics because it has been highly successful in elementary grades for reading, music, spelling, and arithmetic. On the other hand, secondary teachers have little enthusiasm for this method. They indicate that many factors make it impossible to operate a successful program of group instruction. The fact that classes are generally larger than in the elementary grades and that each teacher has five or six classes every day make it an exhausting experience. Besides there is the question as to the effectiveness of group instruction and how it will be accepted by the student. Thus, it would seem important that research be conducted to determine the relative effectiveness of group instruction. As teachers of mathematics, with problem solving an accepted objective, we need to apply research techniques to the solving of our professional problems. It seems unwise to accept the validity of any learning principle on the basis of the verbal statements of a few students, the general impressions of a few teachers, or the enthusiasm of a few experts.

The experiment reported here is a modest attempt to compare the effectiveness of one type of small group instruction with regular class instruction. In one situation the class was divided into small groups and each group was taught separately in the regular classroom. In the control situation the entire class was taught by the regular teacher in the traditional manner. It is hoped that the experimental procedure described here may serve as a pattern for other experiments on this subject.

The teaching of new number bases was the topic for this experimental unit. This topic seemed uniquely suited for this experimentation for several reasons:

(a) It is relatively new to the students and thus previous background of the students is likely to be negligible.
(b) The learning situation is typical of everyday mathematics in that new symbols and new vocabulary are major factors in learning a new number system.
(c) The learning situations provide abundant opportunity for emphasis on meaning.

(d) The role of practice is clearly evident in learning to compute with new numbers.
(e) The teaching of new number bases is advocated as a topic that should be introduced into the secondary school for teaching the understanding of numbers and number processes.

Advanced algebra classes in the University High School at Minnesota were used for this experiment. Two experimental classes and two control classes were selected by stratified random sampling of 48 students. The students were divided into an upper and lower strata according to achievement in previous mathematics topics. An experimental and a control section of equal size were randomly selected from each strata using a table of random numbers. As a result of this sampling there was an experimental and a control class each with 13 students and an experimental and control class each with 11 students. Then each experimental class was again divided into two sub-groups for group instruction. Although these classes were small, they approached the size of the advanced algebra class of a typical high school.

In all the classes each lesson was divided into two 25-minute periods. One 25-minute period was used for instruction and the second 25 minutes for supervised study. All classes used the same reading material, the same learning activities and the same problems. Students spent the same amount of time on worksheets and were not permitted to take them out of the classroom. All students included in the data had the same number of lessions. The highly significant teacher factor was controlled by having each of the two instructors teach one experimental and one control class.

In the experimental sections, the sub-groups were taught by a student who was a member of that group. Each day a different member of the group acted as a student teacher for his group. Students were selected as instructors according to the teacher's judgment of their ability to handle the topic. The more capable students were selected to teach the more difficult topics. Each day the group instructors for the next day were prepared for their teaching by the regular teacher. This preparation was done during the 25-minute supervised study time of each regular class period. The group instructors were allowed to spend an additional 25 minutes outside of class to prepare the lesson they were to present to their group. When groups were being taught by students, the regular teacher observed the instruction, answered occasional questions, and corrected gross errors. This active participation of students as group instructors would seem to be an effective way of stimulating learning.

In the control classes the instructor taught the class as a whole

for 25 minutes. The remaining 25 minutes were used as a supervised study period to complete worksheets. Each 50 minute lesson in experimental and control classes dealt with the same content. Each class used the same worksheets during the supervised study period and spent an equal amount of time on them. The following is the content taught in the experimental unit:

1. The meaning of number words such as digit, integer, number, figure, and number base.
2. The meaning of place value in any number base and counting in base 12, base 5, and base 2.
3. Converting from one base to any other base.
4. The meaning and use of new number vocabulary and symbols in the duodecimal system, e.g., dek for X, el for E, dozen for 10, one do-one for 11, three-gro two-do-five for 325.
5. Computing in base 12, base 5, and base 2.
6. The meaning of duodecimals and the way to change fractions and linear measurements to duodecimals.

This content was taught during six class periods. On the seventh day a 75-item achievement test on number systems was given. This examination included a large number of items using numbers in the base 6 with the idea that exercises in a new number base would be the best way to measure the degree of understanding of number systems. The items were numbered in the base six and test results reported to the class in base six numbers. This proved to be quite interesting.

Prior to the experiment all students were administered an arithmetic understandings test. Intelligence quotients from Stanford Binet tests were available from the personnel files.

(*See test items on the following pages.*)

The mean post achievement score of the pupils in each class was used as the measure of effectiveness of instruction in this study. These means were compared using the analysis of variance and covariance.

When one is dealing with randomly assigned individuals an analysis of variance can be justified in a logical sense. That is, initial difference in ability, knowledge, and motivation are presumed to be equaled in all groups by the random process. Thus, one is justified in comparing the groups by running an analysis of variance on the post test scores. However, it would seem desirable to provide added precision by using covariance to control one or more initial measure such as mental ability and/or arithmetic understandings. This latter procedure is to be preferred over the coarser comparison provided by the analysis of variance.

The homogeneity of the variances was tested by the Welch-Nayer

(*Continued on page 613*)

The following are test items from the arithmetic understandings test: Select the correct answer for each of these questions.

8. The multiplication of 24 by $3\frac{1}{2}$ is the same as 24 multiplied by 3.5. Why do we not indent the partial product 72 in the second example as is done in the first example?
   (a) The multiplier 3 has a different place value in each example.
   (b) The multiplier .5 and $\frac{1}{2}$ are not equivalent.
   (c) The fraction $\frac{1}{2}$ does not have place value.
   (d) The multipliers 3 and $\frac{1}{2}$ have the same place value.

24	24
3.5	$3\frac{1}{2}$
120	12
72	72
84.0	84

10. Look at the division example $(3123 \div 23)$ at the right. What quantity is really subtracted when 23 is subtracted from 31?
    (a) 23 from 31
    (b) 1 from 3 and 2 tens from 3 tens
    (c) 2300 from 3100
    (d) 2300 from 3121

$$\begin{array}{r} 135 \\ 23\overline{)3121} \\ \underline{23} \\ 82 \\ \underline{69} \\ 131 \\ \underline{115} \\ 16 \end{array}$$

Select the largest number in each of the following groups of numbers.

42.	(a) 11010	(b) 10101	(c) 10110	(d) 11001
43.	(a) 2/3	(b) 5/8	(c) 3/4	(d) 6/10
44.	(a) .3	(b) .23	(c) .2431	(d) .301
47.	(a) 3×10	(b) 10^3	(c) 3^{10}	(d) $\sqrt{10}$
48.	(a) $\frac{1}{2}\%$	(b) .5%	(c) .005	(d) .05

The following are test items from the number system achievement test: Mark the true statements plus (+) and the false statements zero (0).

11. The fraction "$\frac{2}{3}$" has the same value in the dozen system as in the ten system.
22. The measurement 3 feet 7 inches can be written 3:7 feet in the duodecimal system.
23. When we borrow in the dozen system in a problem such as $154-2x$ we actually borrow ten units.
25. In the six system, $4+3=11$.
32. In the number base 6, the fraction $\frac{3}{4}$ is equal to 13/20.

Compute in the base 6.

140. Add 342
154

141. Subtract 530
42

142. Multiply 34
5

143. Divide $4/\overline{144}$

Use the following number symbols to solve problems 155–203. □ is 0, 1 is 1, ∨ is 2, △ is 3.

155. Write the next three numbers of this sequence, 1, ∨, △, 1□, 11, 1∨, —, —, —.

200. Add 1□∨
∨△

201. Subtract ∨△□
∨1

202. Multiply 1∨
△

203. Divide $△/\overline{∨1△}$

The results of the tests described above are summarized in Table I below.

TABLE I

SUMMARY OF MEANS AND STANDARD DEVIATIONS ON MENTAL ABILITY ARITHMETIC UNDERSTANDINGS, AND POST ACHIEVEMENT TEST ($N=47$)*

	Experimental Group			Control group		
N	A	B	Combined	C	D	Combined
	11	12	23	11	13	24
Mental Ability						
Mean	118.82	128.67	123.74	118.46	119.82	119.14
s.d.	25.39	15.63	20.85	12.44	13.09	12.74
Arithmetic Understanding						
Mean	43.73	49.00	46.36	47.54	44.91	46.22
s.d.	7.96	7.87	7.92	5.71	6.17	5.93
Post Achievement						
Mean	45.55	48.42	46.98	45.77	47.73	46.75
s.d.	11.13	15.38	13.52	12.94	7.28	10.75

* One individual has been dropped for lack of complete data.

(*Continued from page 611*)

L_1 test. Since the variances were found to be homogeneous at the 5 per cent level, the data were pooled for further analyses.

The significance of the difference between the means of the pooled

data for the experimental and control groups was tested by the analysis of variance as reported in Tables II and III. Probability values of .05 or less were required for rejection of the null hypotheses.

TABLE II

ANALYSIS OF VARIANCE FOR TESTING THE HYPOTHESIS THAT THERE IS NO INTERACTION BETWEEN TEACHERS AND METHODS ($N=48$)

Source of Variation	D.F.	Sums of Squares	Mean Square	F	Hypothesis
Within Groups	44	7119	161.7		
Methods×Teachers	1	1.2	1.2	<1	Accept

TABLE III

ANALYSIS OF VARIANCE FOR TESTING THE HYPOTHESES THAT THE TEACHERS ARE EQUALLY EFFECTIVE AND THAT THE METHODS ARE EQUALLY EFFECTIVE AS MEASURED BY MEAN POST ACHIEVEMENT TEST SCORES ($N=48$)

Source of Variation	D.F.	Sums of Squares	Mean Square	F	Hypotheses
Error	45	7120.2	158.22		
Teachers	1	62.40	62.40	<1	Accept
Methods	1	.12	.12	<1	Accept

On the basis of the results of the analysis of variance summarized in Tables II and III, the following conclusions are stated.

a. The hypothesis that there is no interaction between teachers and methods is accepted. From this it may be inferred that one method wasn't more effective for one teacher at the same time that the other method was more effective for the second teacher.
b. The hypothesis that the two methods were equally effective was accepted. This means that students taught by other students in small groups achieved as well as those taught in classes by the regular teacher.
c. The hypothesis that the two teachers were equally effective was accepted. This, plus the first conclusion, indicates that the group method described here was used as successfully (or unsuccessfully) by both teachers.

The results of this analysis of covariance confirm the conclusions drawn from the analysis of variance. When mental ability and arithmetic understandings are held constant.

a. There is no difference in the achievement of students taught in small groups by another student as compared to the achieve-

ment of students taught by the instructor in a traditional class situation.

b. There is no difference in the achievement of students taught by different instructors, each using the group method or the traditional method.

TABLE IV

ANALYSIS OF VARIANCE FOR TESTING THE HYPOTHESES THAT THE TEACHERS ARE EQUALLY EFFECTIVE AND THAT THE METHODS ARE EQUALLY EFFECTIVE AS MEASURED BY MEAN POST ACHIEVEMENT TEST SCORES UNADJUSTED FOR MENTAL ABILITY OR ARITHMETIC UNDERSTANDINGS ($N=47$)*

Source of Variation	D.F.	Sums of Squares	Mean Square	F	Hypotheses
Methods	1	.055	.055	<1	Accept
Teachers	1	5.832	5.832	<1	Accept
Error	44	545.984	12.409		

* One individual has been dropped for lack of complete data.

TABLE V

ANALYSIS OF VARIANCE FOR TESTING THE HYPOTHESES THAT THE TEACHERS ARE EQUALLY EFFECTIVE AND THAT THE METHODS ARE EQUALLY EFFECTIVE AS MEASURED BY MEAN POST ACHIEVEMENT TEST SCORES ADJUSTED FOR MENTAL ABILITY AND ARITHMETIC UNDERSTANDINGS ($N=47$)

Source of Variation	D.F.	Sums of Squares	Mean Square	F	Hypotheses
Methods	1	.070	.070	<1	Accept
Teachers	1	.271	.271	<1	Accept
Error	42	247.231	5.886		

Although this study involves a short period of time and relatively few students, it does provide a pattern for experimentation. The following attributes of this experiment deserve special mention.

1. The random assignment of individuals to classes and groups illustrates adequate sampling technique. Using a stratified sample assured representation of different ability levels. This randomization adds strength to conclusions based on small samples.
2. Two self-contained experiments, each with its own experimental and control sections, provided replication.
3. The control of variables such as the teacher, content, time, and learning exercises adds precision to the study. This is essential if the effect of the operating factor is to be measured.

4. The analysis of covariance controls variables such as mental ability and prior knowledge. The use of this statistical tool is a substitute for matching individuals.
5. The measuring instrument, with emphasis on understanding, seems appropriate to measure accepted objectives.

To obtain results in which we can have more confidence, this experiment needs to be replicated with the same unit and with other units. Other methods of group instruction may prove to be more effective than this one which used student teachers. To find answers to the many problems we face in improving mathematics instruction, we need extensive research using the statistical analyses illustrated by this experiment. It can be done with the limited resources and time typically available. Try it out in your classroom.

AN INTERESTING DILEMMA: WHY *EXACTLY* 1/2?

JULIUS SUMNER MILLER
El Camino College, El Camino College, California

Consider the following problem: A train is traveling at constant velocity, say 60 miles per hour, on a horizontal track, and it picks up water from a trough between the rails. Say it picks up 12,000 pounds of water in a distance of 1200 feet. Find the *force* to do this.

Solution 1: 60 mph = 88ft/sec. The time to go 1200 feet is 1200/88 seconds. The *time-rate* of gathering water is, therefore, 12,000 ÷ 1200/88 = 880 pounds per second. The force F required of the engine to do this is given at once by Newton's Second Law as

$$F = \frac{d}{dt}(mv) = v\frac{dm}{dt} = (88 \times 880)/32 = 2420 \text{ lbs.}$$

Solution 2: This solution is based on energy relationships. Now do not say, as you are led impulsively to do, that the energy method is risky, because energy is lost! We all know that this system is not conservative and that energy is lost. The astonishing thing is to discover, as we shall directly, that exactly *one-half* is lost! The solution goes as follows: The work done by a force F is equated to the change in kinetic energy of the water. Thus,

$$KE = \tfrac{1}{2}\, mv^2 = \tfrac{1}{2} \times 12{,}000/32 \times (88)^2 = 1.452 \times 10^6 \text{ ft. lbs.}$$

Since this is done in 1200 feet, we write $F = 1.452 \times 10^6/1200 = 1210$ lbs. *And this is precisely one-half that obtained above.*

It is an excellent exercise for beginners, and often for those who are not beginners, to show why it is precisely $\frac{1}{2}$.

WHAT JOHNNY SHOULD READ

Sister M. Stephanie
Georgian Court College, Lakewood, N. J.

Each year in many a college catalog appears a description of a course known as "Readings in Mathematics," or again, "History of Mathematical Thought." It might even be "Development of Mathematics." Perhaps a course by none of these names is offered, but a list of suggested books is published by each department, and the students in a department are expected to read a certain minimum number by graduation. The all-important question is not "What knowledge is the most worth?" but "What books shall appear on the list?" The purpose of this paper is not to settle the question forever more, nor to start a controversy on the merits of these particular books nor even to decide the question of whether there shall be a reading list, but merely to suggest some books which have been found useful for a "History of Mathematics" course. No claim is made for originality of selection; the same books, with variations, have appeared on reading lists before, but no recent compilation seems to have appeared in the literature.

Will all the reading assigned be historical material? Will it be mathematics from original sources? Or modern mathematics from current journals? Perhaps there is a way to achieve a nice balance among all these elements, and then, perhaps there is not, but reading there must be, and some of it must be reading *of* mathematics and not just reading *about* mathematics.

Part of the reading must be historical for the aims of a liberal arts college are defeated if the student is graduated with good technical background but insufficient knowledge of the relation of this information to the culture of the world, insufficient knowledge of when facts were discovered and by whom, what led to these discoveries, the kinds of men who were mathematicians and the times in which these men lived. It was Samuel Butler who said that the best way to study a subject is to study it historically.

A good book for an historical beginning is E. T. Bell's *Men of Mathematics*. Bell gives biographies of about thirty-five great mathematicians in chronological order, so that while his approach is through individual men, he covers the developments taking place during whole periods. His work is journalistic, highly readable, entertaining, containing much of human interest. If anyone had a liver complaint, Bell is not the man to let it pass unremarked! In his *Development of Mathematics*, a companion volume, Bell tries, and successfully, to incorporate with good historical material much technical mathematical information. His professed aim is to give the

undergraduate, not a history book, but an overview of the work accomplished by mathematicians (and still being accomplished—about four thousand research articles are published yearly) so that the beginner in mathematics may know enough about various fields so that he may select that one that is alive and growing rather than one, though interesting *per se*, has become fossilized.

An old-fashioned though highly readable history book, less entertaining than Bell's books, is W. W. R. Ball's *A Short Account of the History of Mathematics*. Individual chapters of this are particularly good for the study of the seventeenth, eighteenth and nineteenth centuries. It is not up-to-date, no reference being made to anything that happened after 1900. Cajori's *History of Mathematics* is good, as is D. E. Smith's two volume work. A standard authority is Heath's *History of Greek Mathematics*, and *The Mathematics of Great Amateurs* by Coolidge is excellent background material. A student should be required to read one book on the history of mathematics, not all of these.

More modern books as Kasner's *Mathematics and the Imagination*, Kramer's *The Main Stream of Mathematics*, and lately Morris Kline's *Mathematics in Western Culture* give the college student an insight into the influence mathematics has had since ancient times and is still having today. They discuss the interrelationships between mathematics and music, mathematics and art, mathematics and nature, and touch in almost popular fashion on modern developments in mathematics. Kline's thesis is that mathematics is a "prime molder and major constituent of our culture." He writes fluently and lucidly and certainly achieves the proof of his thesis, although one may disagree with some of his nonmathematical generalizations. This is a stimulating book.

In this same group should be mentioned *Mathematics, Queen and Servant of Science*, by E. T. Bell which discusses rings, ideals, groups and other algebraic concepts without being a textbook; and Alfred Hooper's *Makers of Mathematics* that describes the beginnings of theory of numbers, geometry and algebra. Hooper's book stops with the seventeenth century. Dantzig's *Number, the Language of Science* tells of the origin of the various kinds of numbers including the transfinite. More difficult reading than any of these, and infinitely more rewarding, is Courant's *What is Mathematics?* This is a book to be owned by every student of mathematics, not borrowed. And if the student lends it, he may have trouble in getting it back!

The philosophy of mathematics should not be neglected in a reading course. A classic, of course, is Whitehead's *An Introduction to Mathematics*. Although it was was written forty years ago, it has not been improved upon. Every student, and especially every student

who intends to become a teacher, should read *Lectures on Fundamental Concepts of Algebra and Geometry* by Young. Wilder's new *Introduction of the Foundations of Mathematics*, perhaps better classified as a text, is a good companion piece devoted to modern mathematics. It is highly informative and its explanations are particularly clear. Much less well-known is Sister Helen Sullivan's *Introduction to the Philosophy of the Natural Sciences and Mathematics* in which several chapters are devoted to the philosophy of mathematics from an Aristotelean and Scholastic point of view. Kershner's *Anatomy of Mathematics* is not easy to read but is an excellent and detailed introduction to the axiomatic method.

Finally, one cannot omit, although one cannot classify so easily, Polya's two delightful books, *How to Solve It* and *Mathematics and Plausible Reasoning*.

What about periodicals? Much good mathematics for the general reader can be found in articles in the *Scientific American:* articles on probability and topology, theory of games and symbolic logic. Many articles, originals and translations of originals, are found in *A Source Book of Mathematics*, edited by D. E. Smith. This contains numerous short articles as they first appeared in periodicals. Now they are, as the joint paper by Brianchon and Poncelet on the nine-point circle, standard textbook material.

Of course one assumes that the undergraduate student be familiar with the journals of mathematics: *American Mathematical Monthly*, *Mathematics Magazine*, and others of his particular interest. If he is preparing to teach he will know of and read carefully *School Science and Mathematics* and the *Mathematics Teacher*. If he does any research at all, mathematical or educational, he will of necessity use these periodicals. Textbooks and books on teaching methods have purposely been omitted from this list. This type of reading is not usually what is included on a reading list, so emphasis has been placed upon those books, some old, some new, all good, without which the embryo mathematician cannot consider himself an educated man. The list is not intended to be exhaustive, merely suggestive. Perhaps the embryo mathematician, at a later stage of his development, will write another book to supplement this list. If it is a good book, there will be no dearth of readers for it.

Books

Ball, W. W. R. *A Short Account of the History of Mathematics*, Macmillan, London, 1915.

Bell, Eric T. *Mathematics, Queen and Servant of Science*, McGraw-Hill, N. Y., 1951.

———. *Development of Mathematics*, McGraw-Hill Book Co., N. Y., 1940.

———. *Men of Mathematics*, Simon and Schuster, N. Y., 1937.

CAJORI, F. *A History of Mathematics*, Macmillan, N. Y., 1919.
COOLIDGE, J. *The Mathematics of Great Amateurs*, Oxford, Toronto, 1949.
COURANT, R. AND ROBBINS, H. *What Is Mathematics?*, Oxford University Press, N. Y., 1942.
DANTZIG, TOBIAS. *Number, the Language of Science*, 3rd ed., Macmillan, N. Y., 1939.
HEATH, T. L. *History of Greek Mathematics*, Clarendon Press, Oxford, 1921.
HOOPER, ALFRED. *Makers of Mathematics*, Random House, N. Y., 1948.
KASNER, E. AND NEWMAN, J. *Mathematics and the Imagination*, Simon and Schuster, N. Y., 1940.
KERSHNER, R. AND WILCOX, L. *Anatomy of Mathematics*, Ronald Press, N. Y., 1950.
KLINE, M. *Mathematics in Western Culture*, Oxford University Press, N. Y., 1953.
KRAMER, E. E. *The Main Stream of Mathematics*, Oxford University Press, N. Y., 1951.
POLYA, G. *How to Solve It*, Princeton University Press, Princeton, 1945.
———. *Mathematics and Plausible Reasoning*, Princeton Univ. Press, Princeton, 1954.
SMITH, D. E. *History of Mathematics*, Ginn and Co., N. Y., 1923.
———. *A Source Book in Mathematics*, McGraw-Hill, N. Y., 1929.
SULLIVAN, SISTER HELEN. *An Introduction to the Philosophy of the Natural Sciences and Mathematics*, Vantage Press, N. Y., 1952.
WHITEHEAD, A. *An Introduction to Mathematics*, Holt and Co., N. Y., 1911.
WILDER, R. L. *Introduction to the Foundations of Mathematics*, Wiley and Sons, N. Y., 1953.
YOUNG, J. W. *Lectures on the Fundamental Concepts of Algebra and Geometry*, Macmillan, N. Y., 1911.

ARTICLES

All of these articles appear in the *Scientific American*.

BELLMAN, R. AND BLACKWELL, D. "Red Dog, Blackjack, Poker," Jan. 1951, p. 44.
BOYER, CARL. "Invention of Analytic Geometry," January, 1949, p. 40.
CARNAP, RUDOLF. "What Is Probability?" Sept., 1953, p. 128.
COHEN, I. BERNARD. "Isaac Newton," Dec., 1955, p. 73.
HAHN, HANS. "Is there an Infinity?" Nov., 1952, p. 76.
———. "Geometry and Intuition," April, 1954, p. 84.
HERWITZ, PAUL S. "Theory of Numbers," July, 1951, p. 52.
HURWICZ, LEONID. "Game Theory and Decision," Feb., 1955, p. 78.
KLINE, MORRIS. "Projective Geometry," Jan., 1955, p. 80.
MORGENSTERN, OSKAR. "Theory of Games," May, 1949, p. 22.
NEWMAN, JAMES R. "The Rhind Papyrus," August, 1952, p. 24.
———. "Laplace," June, 1954, p. 76.
———. "Euler and the Koenigsberg Bridges," July, 1953, p. 66.
PFEIFFER, JOHN E. "Symbolic Logic," Dec., 1950, p. 22.
REID, CONSTANCE. "Perfect Numbers," Mar., 1953, p. 84.
TUCKER, A. W. AND BAILEY, H. S. "Topology," Jan., 1950, p. 18.
WEAVER, WARREN. "Statistics," January, 1952, p. 44.
———. "Probability," October, 1950, p. 44.
WHITTAKER, SIR EDMUND. "Mathematics," Sept., 1950, p. 40.

More pupils are attending high school than ever before in our Nation's history, and more students are staying in school to graduate. Ten years ago 78% of young people 14 to 17 years of age were enrolled in school. Today 87% in that age group are enrolled.

ELECTRICAL AND MECHANICAL OSCILLATORS

A PHYSICAL AND MATHEMATICAL COMPARISON

J. O. Perrine
36 Marion Road, Upper Montclair, N. J.

The least common denominator in practically all forms of communication is a wave. This fact is true whether the system be visual, acoustic or electrical. A wave embodying a parcel of energy is the basic carrier of information.

In some systems of communications, the wave is a mechanical wave like a sound wave. The sound wave in air is a carrier wave for noise (tom-toms and whistles), speech, and music. Light waves are very common carrier waves for information of various kinds. Electromagnetic waves, of which light waves are one example, bear information of a wide variety: telegraph, telephone, facsimile, and television. The waves may be guided by wires or they may go on their own. Electric waves are truly the modern Mercury, who was the god of communication in ancient mythology.

In a broad sense, communication in all cases, except carrier pigeons and letters, books, flowers, candy, etc., where a ponderable and physical object is involved, is a matter of waves, waves, waves. In electrical communication systems, the particular wave is an electromagnetic wave which may be guided by connecting wires or may go on its own unguided path through space. The electromagnetic waves of wire and radio systems are neither animal, vegetable nor mineral. They are imponderable and invisible and do not affect any of the five senses of a human being.

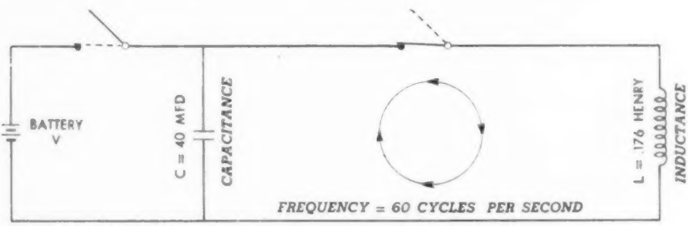
Human throats and musical instruments manufacture sound waves. An incandescent lamp manufactures light waves. The microphone and the telephone receiver are not electrical wavemakers; they are wave translators; they are wave modulators and demodulators.

In many modern electrical communication systems, electromagnetic waves are manufactured by an electrical "tool." This "tool" is the oscillating electrical circuit, which involves an interchange of magnetic field energy and electric field energy associated with an inductance and a capacitance respectively.

Electromagnetic Wave Manufacturers

The electrically oscillating circuit is a manufacturer of electromagnetic waves. It is the objective of this article and the apparatus it describes to discuss the *how* and the *why* and the *how much* of the electrical oscillator. The apparatus is a mechanical oscillator con-

sisting of a weight suspended on a coiled spring. Such a mechanical oscillator is an excellent analogue of an electrical oscillator.



ELECTRICAL OSCILLATING CIRCUIT
Figure A

MECHANICAL OSCILLATING SYSTEMS: COILED SPRINGS AND WEIGHTS

By use of simple apparatus involving different weights suspended on different coiled springs which oscillate up and down, the young scientist can get a better insight and understanding of the method of operation of electrical oscillators.

The method of operation of an electrical oscillator made up of an inductance and capacitance is analogous in many respects to the method of operation of a coiled spring-and-weight oscillator. It will be argued herein that a coiled spring is similar to a capacitance and a weight is analogous to an inductance. The simple formula giving the frequency of electrical oscillations, namely,

$$\frac{1}{2\pi\sqrt{LC}},$$

has an analogous counterpart,

$$f=\frac{1}{2\pi\sqrt{KM}}.$$

The K in this latter formula is an elastic characteristic (coefficient of compliance) of the coiled spring. The M is the gravitational mass expressed in pounds of the object suspended by the spring. The K would be feet per poundal of force.

A capacitance is a reservoir of electric field energy. This energy might be called potential energy. An inductance is a reservoir of magnetic field energy. This energy might be called kinetic energy. In the electrical oscillator, there is an interchange or translation at a prescribed and easily calculated frequency from one type of energy to the other. A compressed or elongated spring is a reservoir of potential energy. A moving weight is a reservoir of kinetic energy. In

the mechanical oscillator, there is an interchange or translation at a prescribed frequency of one type of energy to the other.

It is again to be noted in the very beginning that the simple formula giving the frequency of oscillation in the electrical oscillator has an exact counterpart in the mechanical oscillator.

The Lightning Flash Is Oscillatory

In many cases in many fields of endeavor, Nature supplies a treasure chest of basic and beneficient phenomenon which man learns to understand and use for his needs, after many years of observation and research.

There is no more ancient, more impressive, more realistic, nor more beautiful phenomenon than the lightning flash. It was so overwhelming that many years had to pass before man got up enough courage to study the lightning flash objectively. Franklin did have the required ingenuity and nerve, and found out some simple basic truths about lightning.

Oscillations of one kind or another have been common phenomena to people back through the ages. A branch of a tree in the wind, the surface of the sea, the string of a harp, the pendulum of a clock, were commonplace examples of vibrations. No one in the early days ever dreamed that the lightning flash was oscillatory; that it was an electrical vibration. Folks probably thought that a bucket or barrel full of electricity dropped out of the sky and fell to the earth with a bang. They could see a bucket full of water fall off a table and go down with a great splash on the floor. They thought, perhaps, that the lightning flash was like the falling pail of water; one big splash.

We now know for sure that the lightning flash is an electrical discharge between highly and oppositely charged clouds or between a cloud and the earth. Like all discharges between oppositely charged objects, the discharge is oscillatory. Many experiments and tests show the validity of this concept.

After many years then, thanks to a number of scientists and their researches, the lightning flask is known to be an electrically oscillatory phenomenon. We know now that the lightning flash produces radio waves of miscellaneous composition that are troublesome, and called static. They particularly interfere with man-made electromagnetic waves in wire and radio systems.

Miniature Lightning Flashes: The Leyden Jar

In 1745 Von Kleist, a German, devised an electrical tool, which by reason of its early use by Cuneus of Leyden in Holland became known as a Leyden jar. The Leyden jar is a capacitor and looks a bit like an everyday glass fruit jar. On the inside is a coating of tin foil and on

the outside is another coating of tin foil. The energy of an electric field can be stored in the glass dielectric of a Leyden jar when the metal coatings are given a charge by a voltage source.

Furthermore, the stored energy can be kept stored for quite a while until man is ready to discharge the capacitor and thereby get back the stored energy. If a high voltage source puts energy into the Leyden jar dielectric of glass, and later the two metal surfaces together are connected by a wire, a husky spark will be produced at the instant when the ends of the connecting wire touch the two metal plates between which the high voltage still exists.

Since electric field energy of a fairly sizable amount can be stored in a capacitor when the capacitance (microfarads) is large and the voltage applied is high, the intensity of a spark or discharge can be made high. The Leyden jar or capacitor does not magically intensify electricity as is sometimes said, but if a capacitance is sizable and the voltage is high, the stored energy is quite sizable at least as far as sparks are concerned. To be sure, the energy is dissipated in a very short interval of time and hence is quite intense.

The Leyden jar is oftentimes called a condenser. This word *condenser* is a misnomer since there is nothing condensed. If the stored energy is released at a very short interval of time then the energy is quickly dissipated in heat and light. A snapping sound results whose intensity startles us as high because sparks are impressive. It is like a tiny lightning flash.

In the years 1750 to 1825, the Leyden jar discharge was probably thought of as a miniature lightning flash as it really was. Perhaps it was thought, however, that there was a single, intense and one-shot splash and flash and then it was all over.

Savary

However, scientists of that day were doing research as they are today. In 1827, Savary published a scientific article in a German magazine in which he recorded his discovery that the discharge of a Leyden jar was oscillatory. He discharged the capacitor through a coil of wire (an inductance) surrounding a steel needle. When the discharge was all over in many repeated trials, the needle was oftentimes left magnetized with one polarity and oftentimes with opposite polarity. The final polarity of the needle was determined by the mere chance of greater magnetization in one direction than in the opposite direction. The chances were different in different trials but in most cases, the needle acquired magnetic polarity.

Joseph Henry

In 1841 Joseph Henry of Princeton University, whose name has

been given to the unit of inductance, rediscovered the same fact by the same technique. Interchange of scientific knowledge was neither easy nor common those days, so Henry did not know of Savary's experiments and discovery made 14 years earlier.

Feddersen

In 1857, Feddersen examined the spark of a discharge of a capacitor in a rapidly revolving mirror. He observed in the rotating mirror a group or series of individual, hot sparks instead of a single continuous spark. The brightness of successive flashes decreased as the energy was dissipated with continuing oscillations.

Paalzow

In 1862, Paalzow modified Feddersen's tests. He discharged the capacitor through a long slender vacuum tube made of glass, called a Geissler tube. The alternating discharge of the energy stored in the capacitor produced a series of flashes of light within the tube containing a bit of gas at low pressure. When one terminal was serving as an instantaneous anode (positive terminal), that terminal was bathed in reddish light. The cathode (negative terminal), an instant later, glowed with a bluish color.

Paalzow also looked at the discharge in a rapidly revolving mirror and saw not only a series of bright images in the tube itself, but also that the terminals were alternately red and blue. All four men proved conclusively that the discharge of a capacitor through an inductance was oscillatory but the latter tests of 1862 by Paalzow were more dramatic and convincing.

Lord Kelvin and Mathematics

After and during the discoveries of Savary, Henry and Paalzow, Lord Kelvin, one of the greatest engineers and mathematicians of all time, put mathematics to work quantitatively (1853) on the qualitative fact of electrical oscillations. Lord Kelvin had an experimentally observed phenomenon at his disposal and a basic fact available for mathematical analysis. With his great talent and knowledge of the techniques of mathematics, he put mathematics to work on the discharge phenomena, and from mathematical principles proved that the discharge of a capacitor through an inductance was oscillatory. He derived the very important and useful formula for the frequency of the oscillations

$$f=\frac{1}{2\pi\sqrt{LC}}\,.$$

Ever since 1853 when Kelvin published his equations, the com-

munication engineer has used this formula to do a quantitative job and to know his frequencies as well as a butcher knows his pounds and ounces.

Frequency = cycles per second

$$1. \quad = \frac{.159}{\sqrt{L\ (\text{Henry})\ C\ (\text{Farads})}}$$

$$2. \quad = \frac{159.2}{\sqrt{L\ (\text{Henry})\ C\ (\text{Microfarad})}}$$

$$3. \quad = \frac{5033}{\sqrt{L\ (\text{Millihenry})\ C\ (\text{Microfarad})}}$$

$$4. \quad = \frac{159.200}{\sqrt{L\ (\text{Microhenry})\ C\ (\text{Microfarad})}}$$

Electrical and Mechanical Systems

Electrical phenomena and principles are particularly difficult to grasp and to sense. A generator, a motor, a vacuum tube, a battery, a conducting wire, a magnet, an incandescent lamp, a flourescent lamp, a photo-electric cell, a piezo-electric quartz crystal, and an oscillating circuit, function in seemingly mysterious fashion. One cannot see the electrons in a vacuum tube, the magnetic field, the voltage of a battery, the current in a simple circuit, or the radio waves emanating from an antenna. One cannot see the electric current coursing back and forth in an oscillating circuit. There are no gears, pulleys, belts, pistons, streams of water, moving projectiles, tuning forks, or violin strings, to be seen and tried out with one's own muscles. There are no moving parts in the electric oscillator but yet it functions as a wave-maker, an *electromagnetic* wave-maker. These waves as such cannot be seen, felt, heard, tasted or smelled.

We live at the bottom of a swirling array of radio waves. Radio and television signals are pelting our skin, our eyes, our ears, our tongue and our nose every second of the day and night, yet we go our way unaware of their existence.

Furthermore, electrical oscillations are so fantastically frequent (thousands of times per second, millions of times per second and thousands of millions of times per second) that one is overwhelmed by the bigness of the number.

If one could have a mechanical system with tangible, ponderable, and visible parts oscillating at a low frequency, then a beginner might have a sort of slow motion picture which would get him off on the right foot in his understanding and appreciation of electrical oscillations.

Analogies in General

To further the understanding of electrical phenomena and how they operate, analogies and comparisons are often helpful. All analogies need to be taken with the proverbial grain of salt. In some cases, the beginner may have more difficulty understanding the analogy than he may have regarding the ideas, facts and principles in their own right. If this is true, then the analogy is a very poor one. An analogy is, by definition, a comparison of fairly well known ideas and operations in one's experience and everyday observation with less well known ideas and operations which have not been in one's everyday experience.

Mechanical Oscillators are Analogous to Electrical Oscillators Weights Supported by Coiled Springs

To get a realistic understanding of an electrically oscillating circuit of inductance and capacitance, the oscillating system of a weight supported by an elastic spring offers a very excellent analogy. It is believed that the analogy will not confuse and will not be difficult to understand. It is thoroughly believed that the analogy is a good one and will add to rather than detract from understanding. See Figure 1.

If a weight suspended from a coiled spring is pulled down a bit and then released, there will be oscillations up and down about an original position of rest or equilibrium.

A picture of several springs and their weights is shown in Figure 2.

It will be the purpose and objective of the discussion which follows to show that this mechanical system is a valid analogy in many respects to an electrical oscillatory system. The simple formula of Kelvin for frequency of electrical oscillation has an exact counterpart in the mechanical system.

The electrical formula is

$$f=\frac{1}{2\pi\sqrt{LC}}.$$

The mechanical formula is

$$f=\frac{1}{2\pi\sqrt{MK}}.$$

The mechanical nature of the M and K factors as they are analogous to L and C are discussed later.

A Coiled Spring—A Mechanical Capacitor

1. Point number one to be stated is:

An electrical capacitor stores energy in terms of electric field. A

capacitor is a reservoir of energy. This energy has the quality or nature of potential energy. This capacitor energy can be kept stored for quite a while and then can be recovered.

A compressed or stretched spring stores energy—a sort of potential energy. This energy can be stored for an indefinite time in the spring and can be recovered later.

2. Point number two to be stated is:

When an electrical capacitor is energized by connection to a battery, the electric current (amperes) in the capacitor is at maximum at the beginning of the charging time and is at zero at the end. The back voltage aroused in the capacitor is zero at the beginning when the current is at maximum. The back voltage acquired by a capacitor is maximum at the end of the charging time when the current is zero. That the current into the capacitor leads the back voltage by 90° is an often repeated and readily understood truth.

When a supported spring (a mechanical capacitor) is suddenly energized (stretched) by some outside force, namely a human being, the "mechanical current" or the velocity of the movable end of the spring is at maximum at the very start when the outside pull is applied. At the very instant of start, the back pull aroused in the spring is zero. As the outside force (the mechanical battery) keeps working, the back pull of the spring increases until the spring is fully "charged" or energized. At the end, the mechanical current (velocity of movable end of spring) is zero, and the back pull (voltage) aroused in the spring is at maximum.

Mechanical velocity is a very valid analogy to electric current. Electric current is really equal to *charge times velocity;* it is really QV: also

$$I=\frac{Q}{T}=\text{amperes}=\frac{\text{coulombs}}{\text{seconds}}\cdot$$

The analogy between an electrical capacitor and a spring in the above particular respects is therefore completely pertinent and valid.

3. Point number three to be stated is:

The capacitance of an electrical capacitor (farad) is the ratio of its electric charge (coulombs) to the voltage (volts) acquired by it due to its acquired charge.

$$C\ (\text{Farads})=\frac{Q\ (\text{Coulombs})}{V\ (\text{Volts})}\cdot$$

The Q in coulombs is *amperes times seconds* ($Q=It$) and is the strain or yield which has been put into the capacitor. The volts is the stress to which the capacitor has not only yielded but also the stress aroused in the capacitor itself.

So one could very validly say:

Capacitance is the ratio between electrical strain and electrical stress. If, at one volt, a capacitor had 100 coulombs of strain, it would be a big capacitor and a compliant and receptive capacitor in which a lot of energy was stored with only one volt of stress involved.

This big capacitor, this very receptive and compliant capacitor, would have a large ratio between strain and stress; coulombs and volts. It would be said to have a large coefficient of compliance (C), usually called capacitance (farads).

Mechanical Capacitance

The "capacitance" or the compliance of a coiled spring is the ratio of elongation, strain, or mechanical "charge" to the back force aroused in it when stretched by an outside force. It is the coefficient of compliance, the coefficient of "yieldability."

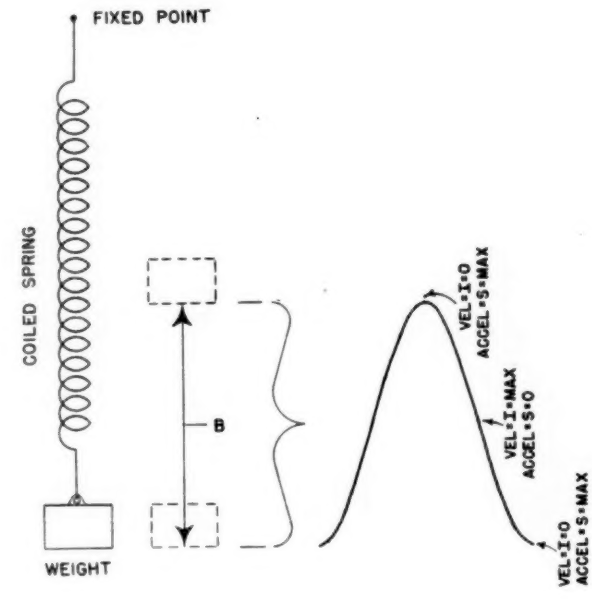


Fig. 1

The mechanical "charge" is the *velocity* (*current*) *times the seconds* involved in the period of charging. The charge is the strain (elongation) in the spring produced by one unit of mechanical force applied.

In mechanics, velocity multiplied by time equals a distance. The coefficient of compliance of a coiled spring is the amount of strain (distance) in feet that the spring stretches when one unit of force (say 1 pound) is hung on a spring. If a spring stretches a lot of feet for one pound of force pulling it, then it would be a big mechanical "capacitor." It would be a very compliant spring. A lot of energy would be stored in it by only one pound of force. The big electrical capacitor with a large coefficient of compliance (capacitance) stores a lot of energy with only one volt involved.

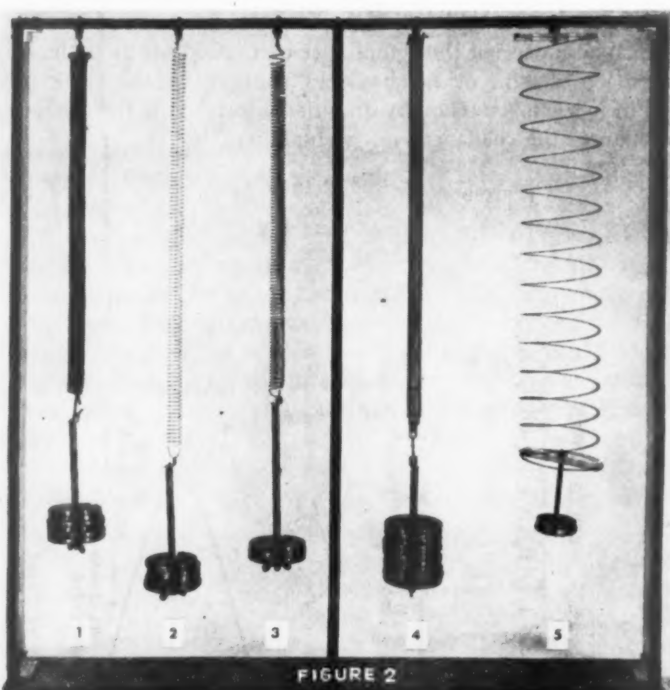


FIG. 2

The spring which stretches a lot of feet and thereby provides a lot of strain per unit of force would not be a stiff spring but would be a compliant and yielding spring.

The ratio of strain to stress or the ratio of elongation to pull, can appropriately be called the *coefficient of compliance* (K) of a spring the coefficient of "yieldability."

The coefficient of compliance of an electrical capacitor is generally

called its capacitance (C) which equals strain/stress which equals Q/V which equals coulombs per volt.

The coefficient of compliance of mechanical capacitor has no particular name but only a symbol K which equals strain/stress which equals feet per pound. It is the reciprocal of coefficient of elasticity or stress/strain in pounds per foot.

Electrical strain and stress in coulombs and volts of a capacitor have their counterpart in feet and pounds of a coiled spring.

The analogy between an electrical capacitor and a coiled spring (a mechanical capacitor) in the above respect is completely pertinent and valid.

Energy in a Capacitor and a Stretched Spring

The analogy between an electrically charged capacitor and a stretched spring is further illustrated when matters are considered from an energy point of view.

The energy stored in the electric field of a capacitor is $\frac{1}{2}CV^2$. If C is in farads and V in volts, then the stored energy would be so many joules or watt seconds.

The energy stored in a stretched spring is given by the same kind of formula, namely $\frac{1}{2}KF^2$. If K is in feet per pound and F, the outside force that provides the stretch of some given amount, is in pounds then the stored energy would be in foot pounds. 1 foot pound equals 1.36 joules which equals 1.36 watt seconds.

Electrical and Mechanical Inductances

4. Point number four to be stated is:

When an electrical inductance is energized by connection to a battery, the electric current (amperes) through it, is zero at the instant of closing circuit and at maximum at the end of energizing time. The back or reactive voltage LS (henries times amperes per second) aroused in it is at maximum at the beginning when the current itself is zero but the time rate of change of current, S (amperes per second), is at maximum.

The electrical charge acceleration (coulombs per second per second) is also at maximum. Amperes per second equals coulombs per second per second.

Since a coulomb equals an ampere second then an ampere is a coulomb per second. Therefore, an ampere per second is a coulomb per second per second. This reminds one of a mechanical acceleration. A velocity in feet per second is analogous to amperes (electric current). An acceleration is time rate of change of velocity in feet per second per second.

The reactive voltage at end of inductance energizing time is zero, but the current is at maximum. The reactive voltage is zero at this time because, even though the current is large and is at maximum, the current is not changing; the amperes per second (S) is zero. The energy stored is a sort of kinetic energy and can be recovered. It is $\frac{1}{2}LI^2$. However, the state of affairs is a dynamic one. The current must be flowing all the time in order that the energy stay stored.

That the electric current driven by the voltage of a battery into an inductance lags the reactive voltage developed in the inductance by 90° is an often repeated and readily understood truth.

When a mechanical mass is given a velocity and thereby energized by an outside force (mechanical voltage), the velocity in feet per second (mechanical current) is zero at the very beginning. Even though the velocity is zero at this time, the time rate of change of velocity (the acceleration in feet per second per second) is likely greatest at this beginning instant. Therefore, the reactive or inertia force (mechanical voltage) aroused in the mass is also greatest at this instant.

The velocity (mechanical current) lags the reactive force (mechanical voltage) aroused in the mass by 90°.

At the end of the energizing process, the velocity of the mass (current) has reached a maximum but the change in velocity (acceleration) is zero. Since the acceleration is zero, the reactive force aroused in the mass is zero. The energy stored ($\frac{1}{2}MV^2$) in the moving mass reservoir can be recovered. As in the case of electrical inductance, the situation is a dynamic one. The mass must keep going and the velocity (mechanical current) must be maintained in order that kinetic energy stay stored.

If attempts are made to stop the moving mass and decrease the velocity (mechanical current) the mass will react and try to keep going to maintain the velocity. There will be a deceleration in feet per second per second and the stored energy will be given back.

$$\text{Magnetic Field Energy} = \text{Moving Mass Energy} = \tfrac{1}{2}MV^2$$

Current in amperes (coulombs per second) can validly be compared with velocity (feet per second). Time rate of change of current (amperes per second or coulombs per second per second) can validly be compared to time rate of change of velocity (feet per second per second).

$$\text{Reactive voltage} = \text{Inductance times rate of change of current} = Ls$$

$$= L\,\frac{di}{dt}$$

Mechanical back force = Mass times acceleration

$$= Ma = M \frac{dv}{dt}$$

Ls is analogous to Ma, $L \frac{di}{dt}$ is analogous to $M \frac{dv}{dt}$

Inductance (1) can validly be thought of as magnetic mass. A mechanical mass (M) in turn can validly be thought of as a mechanical inductance.

SUMMARY #1

Electrical capacitance (C) may be validly thought of as coefficient of electrical compliance. It is a ratio of electrical strain to electrical stress. The farads are the coulombs per volt. Energy can be stored and recovered. The current initially leads the back voltage achieved by the capacitor by 90°.

Mechanical capacitance (K) of a coiled spring is its coefficientof compliance. It is ratio of mechanical strain to mechanical stress. It is the feet per pound. Its mechanical or strain energy (potential) can be stored and recovered. The initial velocity (current) leads the reactive force of the spring by 90°.

Inductance may be validly thought of as magnetic field inertia or, analogously, field mass. It is the henries. Magnetic field energy (kinetic energy) can be stored and recovered. The initial current lags the voltage aroused in the inductance by 90° when an inductance is energized.

Mechanical mass has inertia. Energy can be stored in a moving mass and can be recovered. The mechanical mass has several names and units of measurement; pounds, ounces, etc. The velocity (mechanical current) lags by 90° the reactive force (voltage) of inertia of the mass when a mass is energized by giving it velocity.

C has an analogous mechanical counterpart, namely K.

They are both coefficients of "compliance," of "yieldability."

Inductance (L) has a good analogous mechanical counterpart, namely M, mass.

ELECTROMAGNETIC OSCILLATORS, MECHANICAL OSCILLATORS, FREQUENCY

The electric field potential energy stored in a capacitor and the magnetic field kinetic energy stored in an inductor will interchange and produce oscillations when they are connected in a circuit. These oscillations will have an electrical frequency given by the following formula first provided by Lord Kelvin in 1853.

$$f=\frac{1}{2\pi\sqrt{LC}}$$

The mechanical potential energy stored in a stretched spring and the mechanical kinetic energy stored in a moving mass will interchange and produce oscillations when the mass fastened to the spring is pulled down and released. These oscillations will have a frequency given by the following formula.

$$f=\frac{1}{2\pi\sqrt{MK}}$$

The two formulas not only look alike but are intrinsically and basically exactly analogous. They both are derived by the same fundamental doctrine of energy interchange which in turn involve applied and reactive forces and resulting currents.

Again, in the latter formula, M is the mechanical counterpart of L, and K is the mechanical counterpart of C.

CALCULATION

Since the mass of 15 pounds is to be pulled up against gravity during the oscillations, the so-called absolute unit of force must be used in the K coefficient. The acceleration of gravity (32 feet per second per second) gets into the act, so that the coefficient of compliance in feet per pound really has to be converted into the so-called absolute units. This means that the coefficient of compliance (K) of springs must be expressed in feet per pound AL where the poundal is the absolute unit force. One pound of force equals 32 poundals of force.

$k=1$ foot per 5×32 absolute units of force.

$=1$ foot per 160 poundals.

5 pounds of force $=160$ poundals of force.

$$f=\frac{.159}{\sqrt{\frac{15}{160}}}$$

$=.52$ vibrations per second.

Obviously this is a small frequency because there happens to be a big mechanical capacitance (not a very compliant spring) and a big mechanical inductance, 15 pounds of mass.

But say one had a small mechanical capacitance and a small mechanical inductance. Assume one had a more compliant spring whose coefficient of compliance was 1 foot per poundal. The smaller

mass (mechanical inductance) attached to this spring was .01 pound.

The frequency of mechanical oscillations in the case would be

$$\frac{.159}{\sqrt{.01 \text{ times } 1}}$$

$$= 1.59 \text{ vibrations per second}$$

It should be stated that in this formula for mechanical oscillations the weight of the spring itself is disregarded. So in an actual spring and weight oscillating system, the calculated f will not be exactly the experimentally determined f. In this one single respect, the analogy to an electrical system falls down. A capacitor has no inductance.

The springs and the weights shown in Fig. 2 have the following K's (coefficients of compliance) and M's (masses).

#1, $M = 1.131$ pounds, $K = .0174$ ft per poundal, $f = 1.13$ vib./sec.

#2, $M = .850$ pounds, $K = .0461$ ft per poundal, $f = .72$ vib./sec.

#3, $M = .656$ pounds, $K = .0213$ ft per poundal, $f = 1.34$ vib./sec.

#4, $M = 2.125$ pounds, $K = .011$ ft per poundal, $f = 1.04$ vib./sec.

#5, $M = 1.81$ pounds, $K = .1998$ ft per poundal, $f = .26$ vib./sec.

The periods of a complete vibration (T) for the mechanical vibratory systems in the above table are .88 sec., 1.39 sec., .75 sec., .96 sec., and 3.86 seconds.

Mechrads

It has been pointed out several times that a coiled spring has characteristics which make it analogous to an electrical capacitor. Its mechanical capacitance has been called K. The K has been called coefficient of compliance (yieldability) and is measured by the feet of elongation per poundal of force. The K is analogous to C, the capacitance, in coulombs per volt (farads).

To give K, the coefficient of compliance of a spring, a definite name might be in order. A coined word, "mechrad," might be appropriate. A "mechrad" would be 1 foot per poundal.

The #4 spring has only .0011 mechrads. It is a small mechanical capacitor. Hence, even though the mass (inductance) is 2.125 pounds, it is readily observed by nudging the systems that the #4 frequency is the highest of the 5 oscillatory systems.

Spring and Weight: Frequency

A very interesting "sidelight" on the equation for frequency of a coiled spring mechanical oscillator is worthy of a bit of discussion at

this point. This "sidelight" will summarize the whole matter of mechanical oscillations.

The equation for the frequency is

$$f=\frac{1}{2\pi\sqrt{KM}}$$

Let us consider the KM factor under the square root sign. M is the mass in pounds of the suspended object.

It will be recalled that the K (coefficient of compliance) is the feet per poundal of force. Also that a poundal of force is the so-called absolute unit of force. To get poundals of force one multiplies pounds of force by 32, since acceleration of the force of gravity is 32 feet per second per second.

This gravitational acceleration is a constant, and all objects large or small, heavy or light, fall to the earth with this same acceleration of 32 feet per second per second. This constant is frequently designated by the letter (g). So the coefficient of compliance of a spring is the ratio of feet to poundals.

$$K=\frac{\text{feet}}{\text{poundals}}=\frac{\text{feet}}{\text{pounds times 32}}=\frac{\text{strain}}{\text{stress}}=\frac{l}{Mg}$$

The length, l, in feet is the elongation resulting from a pull of Mg.

Therefore, the factor KM under the square root sign in the formula for frequency becomes

$$\frac{lM}{MG}$$

$$\frac{lM}{Mg}=\frac{l}{g} \text{ since the } M\text{'s cancel out.}$$

With the above reasoning and with the use of algebra, the following formula is derived

$$=\frac{1}{2\pi\sqrt{KM}}$$

in another form, thus:

$$f=\frac{1}{2\pi\sqrt{\frac{l}{g}}}$$

If a weight is hung on a coiled spring and the resultant elongation ("l") of the spring to position of stability or equilibrium is 1 foot,

then this coiled spring system will swing with the same frequency as a simple pendulum 1 foot long. If a heavier weight were used there would be elongation (say 1.5 feet). Then the spring and weight combination would oscillate with the same frequency as a simple pendulum 1.5 feet long. The coiled spring with heavier bob would vibrate more slowly and of course a simple pendulum 1.5 feet long will vibrate more slowly than a simple pendulum 1 foot long.

The equivalent lengths or simple pendulum lengths of the mechanical vibratory systems in previous table are .62 ft., 1.57 ft., .45 ft., .75 ft. and 11.57 feet.

Summary #2

The analogies involved, the formula, the observations of the oscillating springs, the experimental finding of the frequencies and the calculations involving the formulas will afford a lot of good thinking, understanding, and ***knowhow*** and ***know how much***. It is hoped that they will elicit a lot of interest and a bit of fun. One can see the moving mass and observe its varying current (velocity); one cannot see the current in an electrical oscillating circuit.

In electromagnetic systems, it is not easy to get big capacitors and big inductances which oscillate at very low frequencies. In fact, in communication one does not desire low frequencies. One wants to have frequencies of a few thousand, many thousand; many, many thousand and, of course, millions and many millions of cycles per second in micro-wave systems. Kilocycles and megacycles are the vogue in electrical communication—in contrast to the 25 and 60 cycles per second in a-c power system.

This means that capacitors and inductances are electrically and physically small. Therefore, electric communication equipment is small in physical size. In modern times, communication is getting smaller and smaller since higher and higher frequencies are desired and necessary.

Finally, it will be worth while in the light of the ideas previously presented in this article to point up the train of happenings in the mechanical oscillator as it is analogous to an electrical oscillator.

As shown in the accompanying sketch, the body (W) has been pulled down to lowest point by an outside force and then is released. The point (B) is the equilibrium or stable position of the system when it is not oscillating. In other words, the attached weight (W) pulls the spring down as far as (B) and there it would be in a state of rest. The back tension tension aroused in the spring in this case is equal to the weight (W).

After W is pulled down by an amount $A/2$ (one-half amplitude) to its lowest position by an outside energizing force and the spring

has been stretched more than sheer weight stretches it, its upward velocity at instant of release is zero. Since mechanical velocity is nicely analogous to electric current, one can say that the "current" is zero at this instant. The body has no velocity and therefore has no kinetic energy. The entire energy stored in the system by the applied outside force (voltage) is, at this instant, stored in the stretched spring. The stretch or elongation due to the outside force is $A/2$. In other words, all the oscillator energy is stored in the mechanical capacitor. This maximum stored energy (available for interchange) is equal to $\frac{1}{2}KF^2$. In this simple formula, K is the coefficient of compliance of the spring and F is the outside force (voltage) that initially holds the body at its lowest point and against which the equal force aroused in the spring by the outside force pulls back. One is reminded of the energy stored in a charged capacitor being $\frac{1}{2}CV^2$. The two formulas for stored energy are analogous.

At the instant of release and beginning of oscillations, the body (W) is pulled up by the spring. The mechanical reactive force of the mass (M) is greatest at this instant. Even though the velocity is zero at this beginning instant, the increase of velocity or acceleration is greatest. The maximum reactive force involved is mass times acceleration (pounds times feet per second per second); $F=MA$.

The reactive force (voltage) aroused in the body has "led" the velocity (current) at this beginning instant of time.

As the body continues to be pulled up by the force in the spring, the velocity increases until the point of maximum velocity is attained at halfway point (B). This position B is the equilibrium or stable point which the body had due to the sheer weight of the body before an outside force started oscillations.

The velocity of the body has increased up to point B, but it has increased at a decreasing time rate (the acceleration has decreased). The acceleration at midpoint is zero. Since the mass is now moving upwards at its maximum velocity (current) it has its maximum stored kinetic energy, $\frac{1}{2}MV^2$. It has been supplied this energy by the energy of the stretched spring. There has been an interchange of energy.

As soon as the body gets above the midpoint B, the tension in the spring is less than the sheer weight of the body. The body experiences a downward force as far as oscillations are concerned. It might be said that it no longer experiences an upward pull greater than its own weight, so it tries to keep on going and trying to maintain its velocity (current). At this instant, there is no stored oscillatory energy in the spring (mechanical capacitance). The spring has given it all to the moving body. Now the body, moving with maximum velocity (current) and having maximum kinetic energy, starts to decrease in

velocity and give back to the spring the energy it originally received from the spring.

Since there is a downward force on the body moving upward beyond midpoint, the body decreases in velocity and suffers a deceleration. Not only does the velocity decrease but, as the body moves upwards, the velocity decreases at an increasing rate. Finally, the body comes to rest at topmost point and has no kinetic energy. At this instant, the velocity is zero but the deceleration is at maximum.

The reactive force of the body was zero when it was passing upward at midpoint *B* with maximum velocity since its deceleration was zero at that instant. At topmost position, the mechanical reactive force (voltage) is at maximum since its velocity is zero but its deceleration is at maximum. This maximum reactive force at topmost position is equal to maximum reactive force at bottom position.

An opposing reactive force has been aroused in the spring all the time the body was moving from midpoint *B* to topmost position. This reactive force aroused in the spring is at maximum at the top of the vibratory swing and is equal to the maximum mechanical reactive force of the body at this same instant. In fact, the reactive force aroused in the spring is equal to the mechanical reactive force at each and every instant of time.

The body, momentarily at rest at the topmost position, has no kinetic energy. It has given and interchanged its energy to the spring which is compressed or "squeezed" as far as oscillations are concerned.

$$\tfrac{1}{2}MV^2 = \tfrac{1}{2}KF^2$$

When the condition as described in the preceding paragraphs is attained, then things begin to happen in reverse. The compressed spring, compressed as far as oscillations are concerned, pushes the mass down and so oscillation continues. The current (velocity) is of course opposite in direction during the second half of the cycle. After the first cycle, other cycles occur in like fashion.

Simple Harmonic Motion
Sketch and Curve

In all vibrations and oscillations, the back and forth motion is of a very particular and specific nature. If the reader will review the previous statements regarding the motion of the weight as it moves from bottom to top, regarding velocity acceleration, he will be acquiring information regarding what is called simple harmonic motion. A pendulum swings back and forth with S.H.M. A harp string moves with S.H.M. The voltage and current in an a-c power system rise and fall in S.H.M. fashion.

The electric current in the inductance—capacitor oscillator rises and falls in S.H.M. fashion.

Alongside of the accompanying sketch of the mechanical oscillator is a curve, a S.H.M. curve. On the curve are notations regarding current (coulombs per second), its mechanical analogue-velocity (feet per second), the time rate of change of current *S* (coulombs per second per second) and its mechanical analogue-acceleration (feet per second per second). These notations provide the specific characteristics of S.H.M.

If, while the mechanical oscillator is working, the whole affair could be moved from left to right with uniform linear velocity, the weight would make a curve like that shown. This curve is a S.H.M. curve and is the result of two simultaneous motions at right angles to one another; one motion is S.H.M. oscillatory and the other motion is uniform linear velocity. Such curves are also called sinusoidal curves or just plain sine waves.

In the whole process of the mechanical oscillator as described, the stored energy of a compressed spring has interchanged with the stored energy of a moving mass. Oscillations were thereby possible.

So in an electrical system of a capacitor and an inductance, two types or kinds of stored energy are interchanged. In this latter system, electrical waves are manufactured for the purposes of communication for wire and radio carrier systems. In both cases, energy loss due to resistance has been small.

The frequencies of the electrical and its mechanical analogue are readily calculated by a simple formula. The two formulas are analogous. The method of operation of the mechanical oscillatory system permits visual and plausible physical analysis which may help in providing an understanding of the electrical oscillatory systems.

NAVY'S MOST POWERFUL SEA RADAR

The most powerful radar set afloat spots planes 400 miles away. The shipborne "seeing eye" is installed aboard the cruiser Northampton.

Heart of the set is an electronic tube known as a magnetron and tabbed "Big Maggie." At peak power, the tube delivers over 10,000,000 watts or enough power to take care of the electricity needs of a city of 25,000 people.

The core of the tube is a cathode that operates at temperatures up to 3,100 degrees Fzhrenheit, or hotter than molten iron.

The core of the tube is a cathode that operates at temperatures up to 3,100 degrees Fahrenheit, or hotter than molten iron.

The radar set was designed and developed by Westinghouse engineers.

The *Home Study Blue Book* listing courses offered by correspondence schools which meet the standards required by the National Home Study Council can be seen at most public libraries and high schools.

LIESEGANG RINGS ON PAPER[1]

WILL S. DELOACH,[2] BOBBYE K. ALLEN[3] AND OSCAR P. CHILSON[4]
Arkansas State Teachers College, Conway, Arkansas

Lenk and Brach[5] described the formation of Liesegang rings on paper that had been soaked in a solution of $K_2Cr_2O_7$ and then treated by dropping on the paper a solution of $AgNO_3$. Deiss[6] described

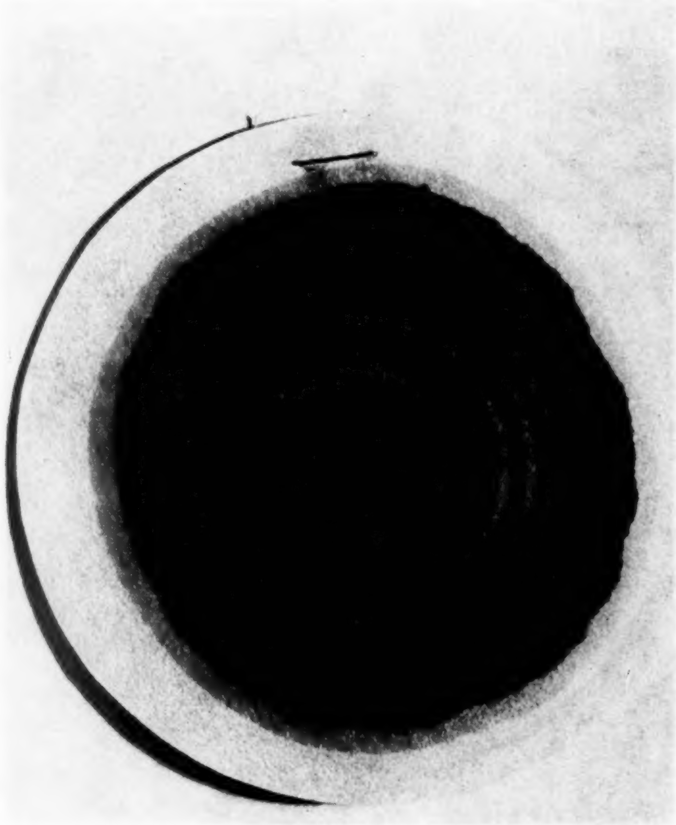


FIG. 1

[1] This paper is based on work done by Bobbye K. Allen and Oscar P. Chilson in a course in Special Problems in Chemistry at Arkansas State Teachers College.
[2] Present address: Catawba College, Salisbury, N. C.
[3] Present address: Department of Biochemistry, School of Medicine, University of Arkansas, Little Rock, Ark.
[4] Present address: Department of Chemistry, University of Arkansas, Fayetteville, Ark
[5] Lenk, E. and H. Brach, *Z. Chem. Ind. Kolloide*, 8, 325-6 (1911).
[6] Deiss, Eugen, *Kolloid-Z.*, 89, 146–61 (1939).

precipitates formed by salts on filter paper, including rings formed when a solution of Mn (II) salt was allowed to drop on filter paper that had been treated with K_2CrO_4 solution and dried. In the work described in the present paper, the rings formed under a variety of conditions by reaction between $FeCl_3$ and $K_4Fe(CN)_6$ solutions on paper were investigated.

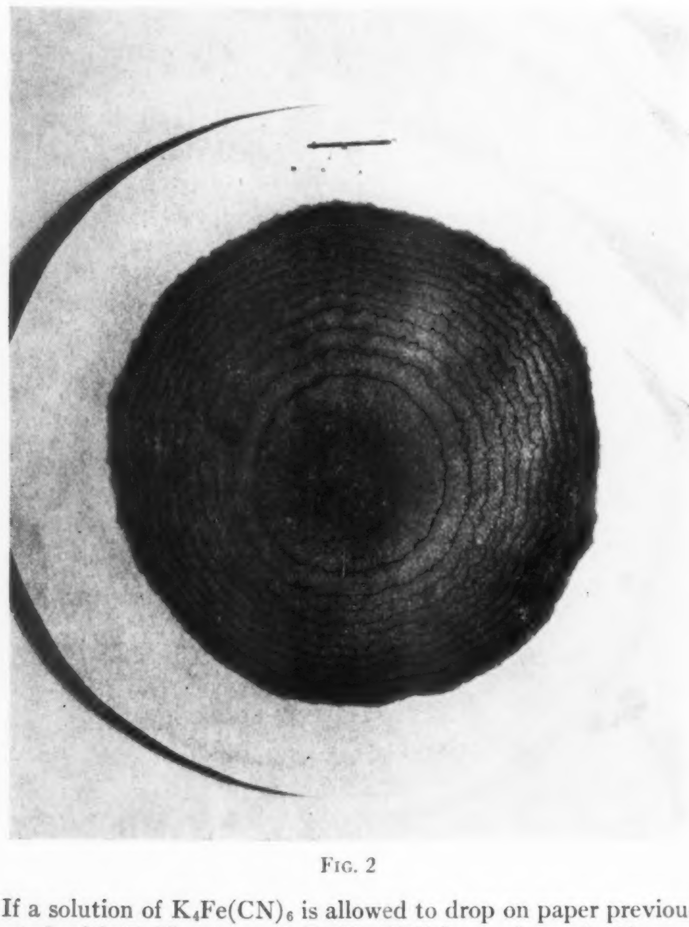


FIG. 2

If a solution of $K_4Fe(CN)_6$ is allowed to drop on paper previously treated with $FeCl_3$, rings of $Fe_4[Fe(CN)_6]_3$ are formed. Rings can also be formed by reversing the order of addition. The formation of rings is affected by the concentrations of the solutions, the rate of dropping, the relative humidity of the air, which reagent is dropped on the other, and whether the paper is wet or dry. To investigate these

factors, one solution was dropped from a burette at a controlled rate on paper previously treated with the other solution. In some experiments the paper was left wet with the solution and in some it was dried, in some cases the paper was left in the open air while the drops were added and in others it was kept in a saturated atmosphere. A suitable container for the saturated atmosphere experiments was a desiccator with some water in the bottom and closed with a piece of

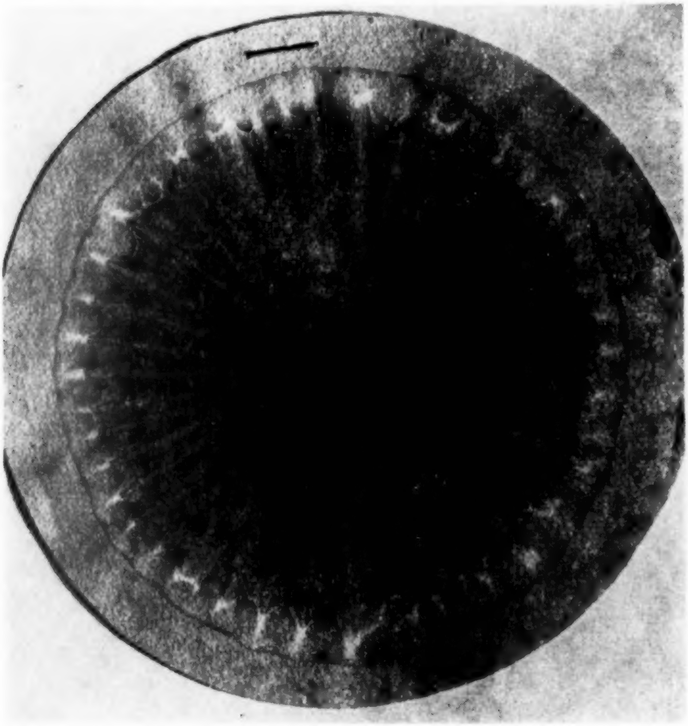


FIG. 3

cardboard that had a small hole for the tip of the burette to pass through. It was found that better and more reproducible results were obtained in a saturated atmosphere, although rings could be obtained in the open air if the humidity was not too low. All of the experiments described here were carried out in a saturated atmosphere.

With $K_4Fe(CN)_6$ dropped on paper treated with 1% $FeCl_3$, dried and placed in a saturated atmosphere, 1 drop every 10 minutes gave better rings than 1 drop every one minute, and 0.1 M $K_4Fe(CN)_6$ gave better rings than either 0.01 M or 0.6 M. Figure 1 shows the

rings formed when 0.1 M $K_4Fe(CN)_6$ was dropped at 10 minute intervals on paper that had been treated with 1% $FeCl_3$ solution and dried before use.

When $K_4Fe(CN)_6$ solution was dropped on wet $FeCl_3$-treated paper in a saturated atmosphere, 0.01 M solution formed some rings when dropped at 10 minute intervals, but none at 1 minute. Very faint

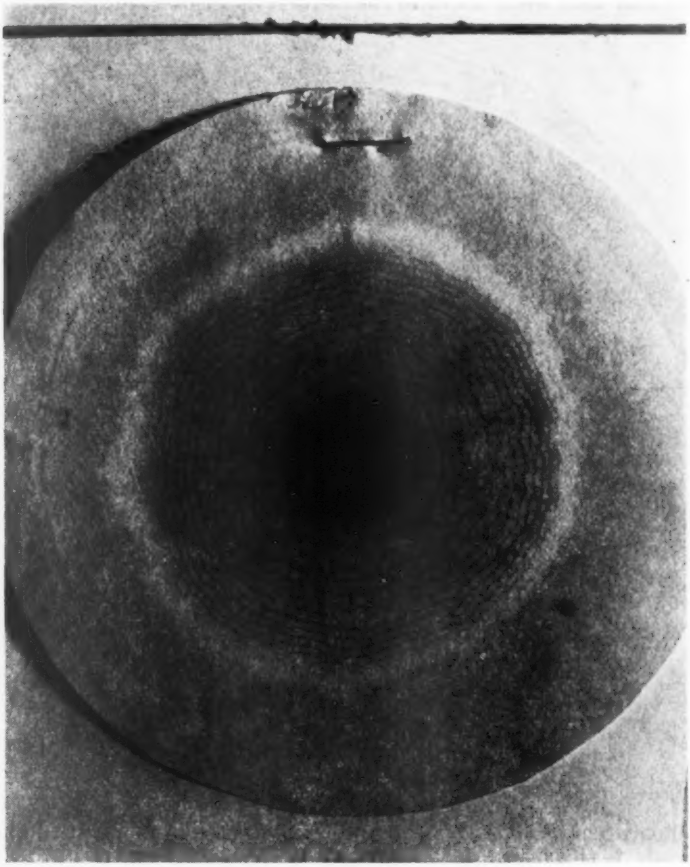


FIG. 4

rings were formed when 0.6 M solution was dropped at 1 minute intervals, with good, distinct rings formed at 10 minutes. Figure 2 shows the latter.

When the reagents were reversed and $FeCl_3$ solution was dropped on dry paper that had been treated with $K_4Fe(CN)_6$ solution, a different set of patterns was formed, although there were still rings.

Radiating from the center were light colored lines, or streaks, with the rings between them. One percent $FeCl_3$ dropped on both 0.1 M and 0.6 M $K_4Fe(CN)_6$, dry paper in a saturated atmosphere, gave good patterns both at 1 and at 10 minutes. Figure 3 shows the pattern formed when 1% $FeCl_3$ solution was dropped at 1 minute intervals on dry paper previously treated with 0.1 M $K_4Fe(CN)_6$ solution. If the paper was left wet with the $K_4Fe(CN)_6$ solution and 1% $FeCl_3$ dropped on it, rings were obtained that were better formed and more distinct and generally without the radiating lines. Figure 4 was obtained when 1% $FeCl_3$ solution was dropped at 10 minute intervals on paper wet with 0.01 M $K_4Fe(CN)_6$ solu-

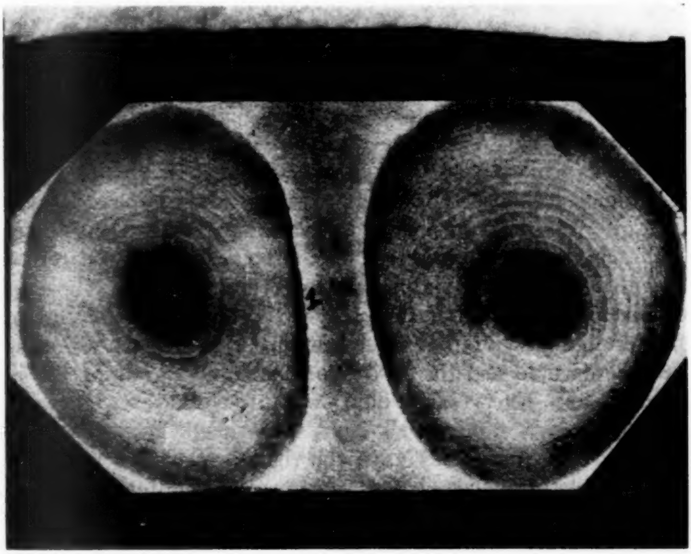


FIG. 5

tion. Distinct rings were formed when the $K_4Fe(CN)_6$ solution was as dilute as 0.001 M and 1% $FeCl_3$ was dropped on at 10 minute intervals. No rings were formed when 1% $FeCl_3$ was dropped at 1 minute intervals on paper wet with $K_4Fe(CN)_6$ solutions over a wide range of concentrations.

Rings can be obtained if $K_4Fe(CN)_6$ solution is applied to a strip of paper, this strip set up as in descending chromatography, and the solvent allowed to drip from the end of the strip on a piece of paper treated with $FeCl_3$. Whatman No. 1 chromatography paper was used both for the descending strip and for the paper treated with $FeCl_3$. One percent $FeCl_3$ was used and the paper was dried. The

$K_4Fe(CN)_6$ solution was applied by making one line across the strip near the lower end, using a capillary tube. About 0.4 M $K_4Fe(CN)_6$ gave better results than either more or less concentrated solution. Average time for a run was about 20 hours. The solvent used was composed of 44% of water, 28% of n-butanol, and 28% of ethanol. Figure 5 shows two patterns formed in this way.

THE THEATER PARTY

BALANCING CHEMICAL EQUATIONS

J. M. KURTZ

7550 Luella Ave., Chicago 49, Ill.

The high spots in an educational career are golden memories. I recall a thrilling experience introducing the balancing of chemical equations with a simple story. A class of high school seniors entered the room as usual. After they were seated I asked them to lay aside their books while I told them a story about a theater party. There was a quick response. They were all agog. Then I spoke.

A quite unusual manager of a theater sent invitations to certain families in the neighborhood to be his guests at a theater party. They all came. The first family arrived bright and early. The manager explained in great detail the elaborate preparations he had made. Only families with a father, a mother and three children were coming. The ushers would seat the parents on the main floor and the children in the balcony. Everybody would walk into the theater by twos.

Could the first family walk right in, I asked? No, was the quick response. Why not? They gave the right answer. Then how many families had to be there before any one could enter? Two, came a loud chorus. The entire class was participating. That was what I wanted. Stepping to the blackboard I wrote FMC_3 explaining that this formula represented a family, F the father, M the mother and C a child, the subscript 3 indicating three children in each family. Where do I place the 2 for two families? In front of the F, they said. I wrote $2FMC_3$ and asked, does this mean 2 fathers 1 mother and 3 children? No, they replied, somewhat chagrined.

After the two families were seated in the theater FM represented a group of parents. Using C for a child where do I put the 2 for a group of children? After the C, was the answer. How many groups of children? Three, they said. The class seemed to enjoy this procedure as much as I did. I wrote only what they told me. This was the result.

outside the theater in the theater

$$2FMC_3 \rightarrow 2FM + 3C_2$$

$2F$	$2F$	
$2M$	$2M$	$6C$
$6C$		

The regrouping as they entered the theater was indicated by placing an arrow after the original group. A plus sign between the last two groups showed the result after the regrouping. Now came the application.

Pure oxygen can be prepared in the laboratory by heating potassium chlorate. The oxygen gas consists of molecules of two atoms each. The molecular formula for oxygen is O_2 and must always be indicated by its formula in a chemical equation. But when oxygen is combined with other elements like potassium and chlorine we must use the formula for that compound. The formula for potassium chlorate is $KClO_3$. When this compound is heated all the oxygen is separated from the other elements and a solid, potassium chloride, KCl, is left. The equation for this chemical change is very similar to the equation for the theater party. Let us compare them.

theater party $2FMC_3 \rightarrow 2FM + 3C_2$

chemical equation $2KClO_3 \rightarrow 2KCl + 3O_2$

In the theater party we count persons. In the chemical equation we count atoms. Each atom is recognized by its symbol. For the beginner in chemistry the great variety of symbols is confusing. It would be much simpler to write the chemical equation $KClO_3 \rightarrow KCl + O_3$ but it would be incorrect. The oxygen formula must be written O_2. The chemistry student must learn to write the correct formulas in writing equations. And he must learn to count atoms.

We can write three different equations for nearly every chemical reaction: (1) the word equation, (2) the skeleton equation, (3) the balanced equation. Sometimes the skeleton equation is already balanced. In (1) we name every substance involved in the reaction. In (2) we write the correct chemical formulas for the substances named in (1). In (3) we observe first if the number of atoms in the newly formed substances is the same as the number of atoms in the original substances. If they are the equation is balanced. If not the first temptation for the beginner is to change subscripts. That temptation is greatest in an examination. The correct procedure is to write coefficients before some or all of the formulas in the skeleton equation so that the number of atoms in the newly formed substances will be the same as the atoms in the original substances. A simple check can be made by drawing a vertical line beneath the arrow in the equation, writing the symbols on both sides of the line and the

number of atoms after each symbol. We can use potassium chlorate to represent the three types of equations and the check.

Word equation	potasium chlorate→potassium chloride+oxygen
Skeleton equation	$KClO_3 \rightarrow KCl + O_2$
Balanced equation	$2KClO_3 \rightarrow 2KCl + 3O_2$

	K2	K2
check	Cl2	Cl2
	O6	O6

It will be observed that the potassium and chlorine were already balanced in the skeleton equation. The oxygen alone required the balancing of the equation. Now recall the theater party where the first family waited for the arrival of the second family before any one could enter the theater. That was due to three children in the families. Or in other words the second family balanced the equation. That story illustrates the basic principles involved in balancing chemical equations.

A DEMONSTRATION ON THE ELASTICITY OF GLASS, THE INCOMPRESSIBILITY OF WATER, AND THE COMPRESSIBILITY OF A GAS

Julius Sumner Miller
El Camino College, El Camino College, California

Fill a flat-sided bottle (like a whiskey bottle) with water and allow a large air-bubble to be trapped. Stopper tightly. A screw-cap is best. With the bottle on its long, narrow side the air bubble now behaves as the bubble in a capenter's level. Note carefully its size. Now grasp the bottle on its *wide* sides and squeeze. The bubble is compressed very noticeably. If the bottle is squeezed on its ***narrow*** sides the bubble expands.

More people enroll each year in private correspondence schools than enter the freshman classes of all colleges and universities in the nation. A list of accredited correspondence schools can be seen at your public library or high school, or it can be obtained from the National Home Study Council, 1420 New York Ave., N.W., Washington 5, D.C.

One of every four persons in the United States is attending school or college this year.

MATHEMATICS INSTRUCTION IN THE SECONDARY SCHOOLS OF NORWAY

Howard F. Fehr,
Teachers College, Columbia University

with assistance of Kay Piene, Rektor
Pedagogical Seminary, Oslo, Norway

This paper give's a general over-all view of the educational program, the specific mathematics program of the Realskole and Gymnas, and a brief description of the training of teachers of mathematics for instruction in the secondary schools.

The Educational System

In Norway children begin formal schooling at the age of 7. The elementary schools runs for 7 years with a possibility of 1 additional year in a continuation school for those pupils who will then terminate their formal education. This education is compulsory for all children and in 1956 the elementary schools enrolled 430,000 pupils, with 24,000 additional pupils in the continuation school. In 1956–57 there will be as estimated 30,000 in the continuation schools.

Upon successfully passing the leaving examination (see A on chart) a student may be admitted to the Realskole, which is a two or three year program, or to the Gymnas which is a five year program. These programs begin at the age 13 to 14 years and end at the age 18 to 19 years. Today there are around 50,000 pupils in secondary schools which comprises about 19 or 20% of the population [267,000 (1956)] of this age group.[1] In the classes III, IV, and V of the Gymnas the number of students is 15,000 comprising about 12 to 15% of the total population of this age group.

For the first two years, classes I and II, the same instructional program exists in both the Realskole and Gymnas. There is an examination at the end of the second class for all students. Those who pass may continue either to one additional year in the Realskole, (Class III) or to three additional years in the Gymnas. The instruction is the Gymnas is specialized into four particular lines, the two principal lines being Linguistic (English) and Real (Mathematics).[2]

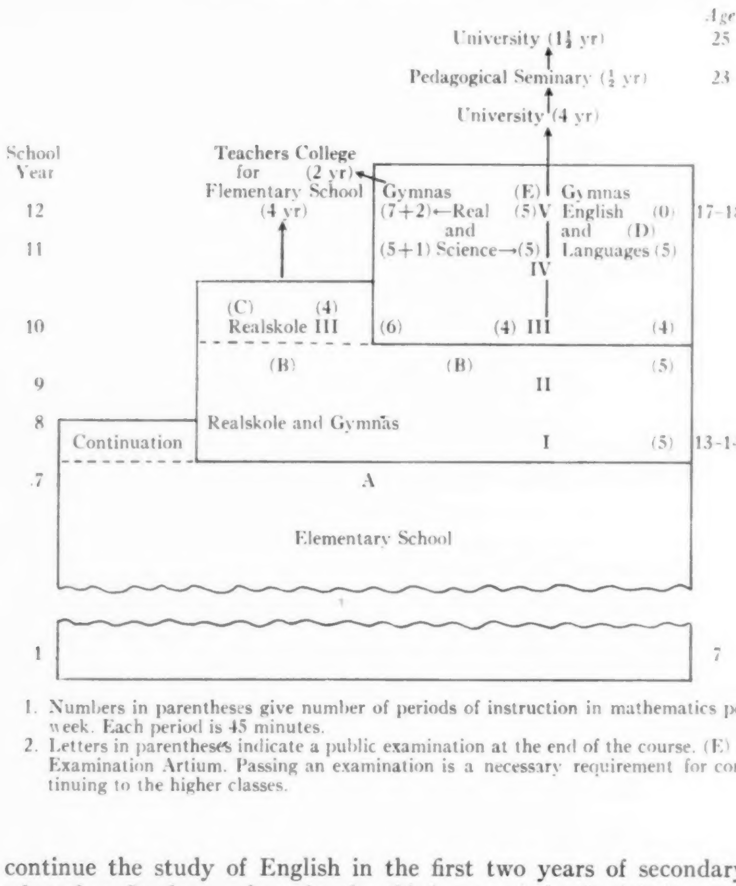
All students who attend the Realskole or Gymnas must if possible[3] have studied English in grades 6 and 7 of the elementary school and

[1] In Oslo more than 46% of this age group is in the secondary school. In Trondelay 12%, therefore, there is great variation from district to district.

[2] Four lines: Latin (very few), English, Natural Science (very few), Mathematics.

[3] All cities and the larger country districts give English training. In 1953 81% of the city pupils and 28% of the country pupils fulfilled the 2 year English course.

THE EDUCATIONAL SYSTEM IN NORWAY
WITH SPECIAL REFERENCE TO MATHEMATICS



1. Numbers in parentheses give number of periods of instruction in mathematics per week. Each period is 45 minutes.
2. Letters in parentheses indicate a public examination at the end of the course. (E) is Examination Artium. Passing an examination is a necessary requirement for continuing to the higher classes.

continue the study of English in the first two years of secondary education. Students who take the third year of the Realskole must take an examination upon leaving and if they pass it, they are eligible to enter a four year training program in teacher training colleges to prepare for teaching in the elmentary school. Students who continue in the Gymnas, upon successful completion of their work as shown by passing examinations, are eligible to enter the University. These students may also elect to enter the Teacher Training Colleges for Elementary School and pursue a 2 year program. It is assumed that the last two years of the Gymnas instruction are equivalent to the first two years in these teacher training colleges.

The Mathematics Instruction

The instruction in mathematics in the elementary school is confined to the arithmetic of computing with whole number, fractions, and decimals, finding areas and volumes of geometrical figures, and solving numerical problems. This is the same instruction that occurs in American schools in Grades 1 to 8. In general the amount of time given to mathematics is 5 periods per week, except in the first three grades, where it is a little less.

The instruction in Classes I and II consists of arithmetic, algebra and geometry. While there is expected to be fusing and interrelating of these subjects they are for the most part taught as separate subjects. These subjects are studied 5 periods per week each year (1 period equals 45 minutes). The instruction in each of these subjects includes the following:

Arithmetic: Percentages, simple interest, permille, business calculations, checks, bonds, stocks, drafts, ratio, proportion, foreign currency, specific weight, averages, constant velocity, areas (of parallelogram, triangle, trapezoid, circle, sector), volumes (of prism, cylinder, cone, and sphere) and problems where the pupil must find out what is known in advance and what must be worked out in order to plan a solution as simply as possible. This emphasis on how to solve problems is directed toward helping students do well on the several examinations they must take. The students are also trained to do rapid calculation and to estimate an approximate value of a solution. The use of tables to find values for n^2, $\sqrt{n}$, and $1000/n$ for $0<n\leqq 1000$ is stressed. Students are not required to find square roots by calculation but may use tables at all times.

Algebra: The work covers all the material that occurs in a standard first course in alegbra in this country with the following additional topics: The role that 0 and 1 play in a number system as identity elements; rules of divisibility for 2, 5, 4, 25, 3 and 9 and prime and composite numbers. This is in fact an introduction to the theory of numbers. There is little emphasis on proof.

Geometry: The work covers all the material found in current popular textbooks in plane geometry in our country with the addition of work on symmetry about an axis and a point. Starting with intuitional knowledge the students build a systematic course with proofs, not in a strictly Hilbert manner, but learning the classical scheme of hypothesis-conclusion-demonstration based on undefined and defined elements and dependent axioms.

The result of this teaching is measured by a public examination given in all Realskole and Gymnas at the end of the second year

(Class II). Those students who complete their schooling in one more year go to the Class III of the Realskole and during this year take a course in "Social Mathematics" occupying 4 periods of instruction each week. This is a new course in the last ten years and is growing in popularity. The two aims of this course are (1) orientation in present day society and in matters of life that are ruled by number and (2) application and theory of mathematical knowledge needed in problems of practical life. The course is very similar to some of our courses in consumer mathematics, only it is far more mathematical. The following is a partial list of the topics treated.

Banks and finance; interest and discount; compound interest and annuities; mortgages; bonds; foreign exchange; stock companies; cooperatives; failures; estate and inheritance taxes; insurance (all kinds); price indices; level of wages and real wages; correlations; indirect taxes; government and municipal budgets; incomes and expenditures; bonds; quotations; nominal, immediate and effective interest rates; graphs; elementary statistics; and elementary algebra and geometry. A public examination is given on this course at the end of the year.

In the upper divisions, classes III, IV, and V, a real distinction is made between classical and English students on the one hand, and science and mathematics students on the other, even though all four of these lines prepare for the university. The linguistic students study mathematics 4 periods per week in Class III, 5 periods per week in Class IV, and not at all in Class V. In general the material studied includes an extended knowledge of geometry and algebra, and mathematics applied to economics and life. The material is essentially that found in a typical American Intermediate (second course) Algebra textbook with the addition of numerical trigonometry of right and oblique triangles, and the elementary mathematics of finance. A public examination is given at the end of the Class IV.

The strong mathematics' programs are those in the Natural Science and Real lines (Mathematics) in Classes III, IV and V. The natural science line includes algebra, extended plane geometry, trigonometry, analytic geometry, elements of differential and integral calculus. This material is integrated and is taught in 4, 5 and 5 lessons per week in each of the years III, IV, and V respectively.

In the "Real Line" mathematics is the strongest program of all subjects. It is taught 6, 5 and 7 periods per week in years III, IV and V respectively. In addition to this the students study a separate one period course in mathematical geography in Class V and another period of descriptive geometry in each of the Classes IV and V. Likewise Physics is studied in both Classes IV and V a total of 6

lessons each week. The essential purpose of the mathematics instruction is to give familiarity with the use of practical and technical problems and some training in the use of insight in mathematical research.

The subject matter taught in the Real line is approximately the following:

Class III (10th year)	Geometry, including homothetic figures and vectors; algebra, including study of functions and derivatives. An introduction to trigonometry.
Class IV (11th year)	Solid geometry, trigonometry, advanced algebra, differential calculus.
Class V (12th year)	Analytic geometry, differential and integral calculus.

The degree of mathematical competence expected at the end of this study is illustrated by the sample 1955 Examen Artium which is appended to this article.

Different requirements hold for teaching Classes I and II, than for teaching Classes III, IV, and V. All teachers are graduates of the Real line Gymnas and of the University where they selected mathematics as one of the three major areas of study. This study includes calculus from a rigorous point of view, differential equations, complex functions, number systems, geometry of matrices and vectors, differential geometry, and some work on foundations of mathematics.

If the teacher intends to teach only Classes I and II and Realskole III, upon passing the University examination he will attend the Pedagogical Seminar for one half year where he will study foundations of education (History, philosophy, psychology, hygiene, administration, curriculum and methods of teaching) and teach in a school. He will then be about 23 years of age and will start teaching.

To teach in Classes III, IV and V, however, he must return to the University after the Pedagogical Seminar and continue his University study for another $1\frac{1}{2}$ to 2 years.[4] This study is entirely in the field of mathematics and includes, theory of functions of a real and complex variable, theory of numbers, abstract algebra, analytic geometry of space, and special fields such as probability, topology, calculus of finite differences, etc. Upon passing examinations and writing a thesis he will be permitted to teach in the Gymnas.

If space permitted it would be enlightening to describe the teaching that goes on in the classes in the Norway schools. It must suffice to say it is excellent, and that the attitude of the students in the Norway Gymnas would be most welcome in most American secondary schools.

[4] In practice a lot of the mathematics teachers in the Gymnas have not taken mathematics as their major subject in the four year University program, but physics, geography etc. Rather few specialize in mathematics.

1955 REAL LINE. EXAMEN ARTIUM. NORWAY

I

In triangle ABC, angle C is 90°, M is the mid-point of AC, AC is of length b and angle $A=x°$, $\angle ABM=v°$.

1) Construct triangle ABC, given b and v. In the construction use $b=7$ cm, and $v=15°$. Find (from the figure) the condition that v must fulfill if the construction is possible.
2) Find tan x expressed in terms of tan v. Discuss the formula you find, and compare the result of this discussion with the result of the discussion of the construction in part 1).

II

In a regular quadrangular pyramid, the side of the base is a and the altitude is x. Show that the radius r in the circumscribed sphere of the pyramid is given as a function of x through

$$r=\frac{2x^2+a^2}{4x}.$$

Find the least value of r and the corresponding value of x. In this case how long is the lateral edge and how large is the inflectional angle between the lateral plane and the base?

In the case where the radius is minimal, a plane is passed through an edge of the base dividing the surface of the sphere in two parts in such a way that these have the ratio: 2:1. Find the inflectional angle between this plane and the base of the pyramid.

III

Find the coordinates for the pole P to the line $y=kx$ with regard to the ellipse $4(x-3)^2+9y^2=36$.

Then find the equation of the locus of the intersection point of the lines $y=kx$ and a parallel to the x-axis through P, when k varies.

The locus is a parabola with the equation

$$y^2=\frac{4}{3}x$$

Compute the coordinates of the intersection points between this parabola and the given ellipse, and draw both curves on graph paper using 2 cm as a unit.

Compute the area of the closed figure bounded by these two curves and lying above the x-axis.

"LOST SOUL'S" LAMENT ASCRIBED TO JUNGLE BIRD

Indians call it "the lost soul," but Smithsonian Institution scientists say the eerie cry comes from a seldom-seen bird related to the North American whippoorwill.

The sorrowful cry rends the night air at the Smithsonian Institution's tropical preserve, raising gooseflesh on visitors. Natives on Barro Colorado Island in Gatun Lake, location of the Panama Canal Zone preserve, call the bird "alma perdida," the lost soul.

The late Dr. Frank M. Chapman, noted ornithologist with the American Museum of Natural History, said he "never heard such a human sound from a brute before." The cry has been described as a "woman's voice, a deep, mellow contralto calling in hopeless grief." A Balboa woman said when she heard it she thought a neighbor's wife was being beaten.

THE PLAN AND ELEVATION OF A CUBE STANDING ON A VERTEX WITH A LONG DIAGONAL VERTICAL, ALSO OF A RECTANGULAR BLOCK IN A SIMILAR POSITION

JOHN SATTERLY
University of Toronto, Toronto, Canada

Plans and Elevation of Rectangular blocks standing on their faces are easily drawn but when the blocks are standing on a vertex even with a long diagonal vertical the problem is harder and calls for much thought. It is well to have models ready to clarify the problems. Such models are easy to come by in the cardboard containers in which many articles are sold. I recommend my readers to take drawing board, T-square, set squares and compasses and to make the diagrams as they read the instructions.

THE CUBE AND BLOCKS

These bodies have 8 vertices, 6 faces and 12 edges. If we denote the number of vertices, faces and edges by N_1, N_2, N_3 respectively then $N_1+N_2=N_3+2$ an example of a general law which holds true for all convex polyhedra and is known as *Euler's Law.*

THE CUBE: GENERAL CONSIDERATIONS AND CALCULATIONS

Denote the cube by $ABCDEFGH$. In our drawing we shall suppose the cube to stand on the ground plane and in front of the vertical plane. Its plan is seen if we look vertically downwards on the cube, the elevation is seen if we look towards the cube and the vertical plane. In other words the plan is the vertical projection of the cube on the ground or horizontal plane and the elevation is the horizontal projection on the vertical plane. The ground line XY (Fig. 1) is the intersection of the horizontal and vertical planes and in order to represent both plan and elevation on a plane sheet of paper we suppose the vertical plane to be revolved backwards around the ground line or (as the draughtsman says) rebatted through a right angle to lie in the same plane as the horizontal plane. The plan and elevation of the vertices of the cube are lettered in small letters undashed and dashed respectively corresponding to the capital letters of the vertices.

Let s = length of edge of cube, the length of a diagonal of a face $=s\sqrt{2}$ and the length of a long diagonal $s\sqrt{3}$. This last gives the height of the elevation of the diagonal.

Examination of a model cube and its symmetry soon convinces one that starting from the topmost vertex A the descent to the bot-

tom vertex H along edges may be made in three equal and equally inclined steps. Thus three vertices are at altitude equal to $\frac{1}{3}(s\sqrt{3})$ and three others at altitude equal to $\frac{2}{3}(s\sqrt{3})$. Since all edges are equally inclined to the horizontal their plans are of equal length therefore the outline of the plan is a regular hexagon.

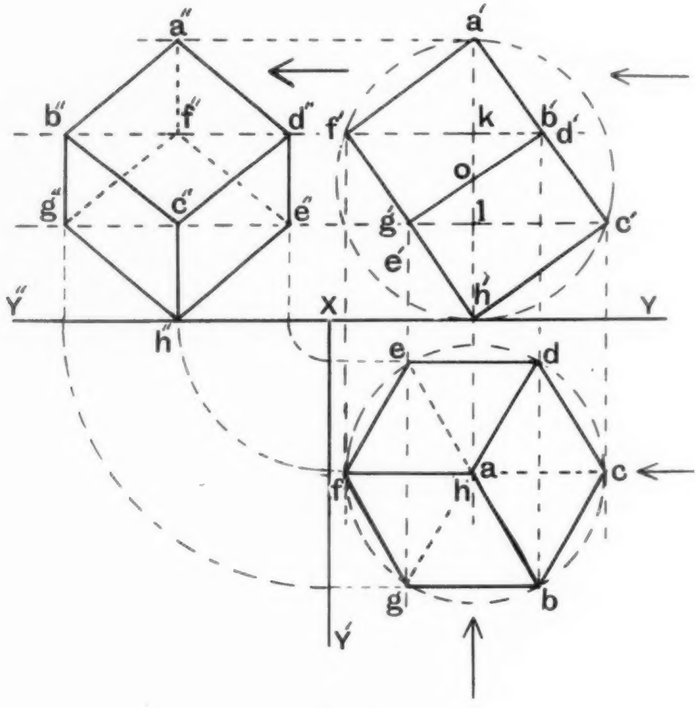


FIG. 1

The Elevation

We start by supposing the cube to have four edges parallel to the vertical plan and shall proceed with the elevation. Draw $a'h'$ perpendicular to XY (Fig. 1) and of height equal to $s\sqrt{3}$, the true length of a diagonal. Bisect it at O and trisect it at k and l. All vertices are at an equal distance from the centre o, therefore describe a circle with o as center and oa' as radius, cutting the horizontal through k in f' on the left and the horizontal through l in c' on the right. Join $a'f'$, $c'h'$, $a'c'$, $f'h'$. Draw $g'ob'$ through o parallel to $a'f'$ (or $c'h'$) cutting $a'c'$, $f'h'$ in g', b' respectively. The elevation $g'b'$ of the edge GB is

right in front of the edge ED so that the $g'b'$ is also lettered $e'd'$. The elevation is now complete. The edges AF, BG, DE, CH are shown in their true lengths.

The Plan

Project $a'h'$ downwards and select a point a as the plan of A at a convenient distance in front of XY. Draw a line fac through a parallel to XY and project down from f' and c' by lines at right angles to XY to cut fac in f and c respectively. The plan of the cube is a regular hexagon, therefore with a as center and ac (or af) as radius describe a circle and in this circle place by use of a 60° set-square, the regular hexagon $bcdefg$. Join ab, ad, af by full lines. The plan of H the bottommost vertex is at h immediately under a. Join hc, he, hg by dotted lines to show that these edges are not seen when vision is vertically downwards. The plan is now complete.

Note that the circle used in constructing the plan is smaller than the circle used in constructing the elevation.

Further calculations

In the elevation $a'c'$ is the full length of the diagonal of a face and $\therefore = s\sqrt{2}$, also $h'c' = s$, $h'l = \frac{1}{3}s\sqrt{3} = s/\sqrt{3}$. $\therefore lc' = s\sqrt{\frac{2}{3}}$ and this is the length of the radius of the circum-circle of the plan and therefore the length of the plan of each edge. In the plan the distance bd also shows the full length of the diagonal of a face and therefore $= s\sqrt{2}$.

The Side Elevation seen looking from right to left as indicated by the left-pointing arrows is the projection of the cube on a vertical plane perpendicular to both the ground plane and the previous vertical plane. Points on this side elevation will bear double dashes.

Draw a new ground line XY' as shown. Project b, c and d to the left to cut XY' and with X as centre revolve the points of intersection around to cut the old XY produced to the left. Erect verticals from these points to cut the leftward projections from a', b', c', $\cdots$, h' and we get the points a'', $\cdots$, h'' on the new elevation. *Or* we may erect a vertical $h''a''$ first and two other verticals $d''e''$, $b''g''$ on the right and left respectively of it at distances away equal to $\frac{1}{2}bd$ and project over horizontally from the old elevation to get the points a'', $\cdots$, h''. The point F is invisible from the right so that $f''a''$, $f''c''$, $g''g''$ must be shown as dotted lines; the other joins must be full. Note that the outline of the new elevation is *not* a regular hexagon; $b''g'' = a''f''$ and this is obviously $< a''b''$.

Other Methods of Approach and Graphical Constructions

1. Fig. 2(1) shows how from a square $PQRS$ of side s the lengths of

a face diagonal and a long diagonal may be obtained graphically by joining PR, drawing RT of length $s \perp PR$ and joining PT.

2. *Suppose the hexagonal plan of the cube were given, how could we find the points in the elevation?*

Upward projections from a, c, f give the breadth of the elevation. We know that

$$\frac{\frac{1}{3}(\text{height of the elevation})}{\text{side of plan}} = \frac{s/\sqrt{3}}{s\sqrt{2/3}} = \frac{1}{\sqrt{2}}$$

So if ed in Fig. 2(2) is a side of the plan describe a semicircle on it, erect a perpendicular mn at the middle point m to cut the semicircle in n and join en; from the construction $en = \sqrt{2}(ed/2)$ and as $ed = s\sqrt{\frac{2}{3}}$, $en = \sqrt{2}(s/2)(\sqrt{\frac{2}{3}}) = s/\sqrt{3} = h'l$. The elevation can now be drawn.

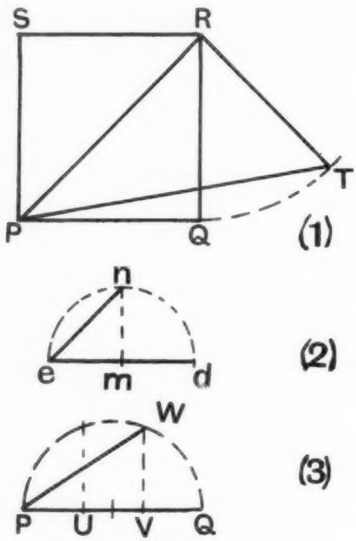


FIG. 2

3. *Suppose the length s of the edge of the cube was given, how can we proceed to the plan without making the elevation?*

Draw PQ, Fig. 2(3), equal to s and on it describe a semicircle. Trisect PQ at U and V. At V erect a perpendicular to cut the semicircle in W. Join PW. It will be the length of the plan of an edge, for

$$WV^2 = PV \cdot VQ = \left(\frac{2}{3}s\right)\left(\frac{1}{3}s\right) = \frac{2}{9}s^2$$

and

$$PW^2 = PV^2 + WV^2 = \frac{4}{9}s^2 + \frac{2}{9}s^2 = \frac{2}{3}s^2$$

$$\therefore\ PW = s\sqrt{2/3} \text{ and this} = ab. \text{ (Fig. 1)}$$

Reduced to decimal fractions we see that

$$a'h' = 1.732 \times s, \quad a'k = kl = lh' = 0.577 \times s$$
$$a'c' = 1.414 \times s, \quad ab = ad = af = 0.821s$$
$$bd = b''d'' = 1.414 \times s$$

Also $a''b'' = 0.913 \times s$ and $b''g'' = 0.577 \times s$ illustrating the disparity of the sides of the profile of the hexagon in the side elevation.

A Rectangular Block Standing on a Vertex with Its Long Diagonal Vertical

We need to know the lengths of certain projections. To get them look at Fig. 3 which represents the block resting on one face $ABCD$. The block has edges a, b, c. Let α, β, γ equal the angles between the long diagonal AG and the sides AB, AD, AE respectively. The projection of AB on a plane perpendicular to

$$AG = a \sin\alpha = a \cdot \frac{BG}{AG} = a\frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}}$$

Similarly for the projections of AD and AE.

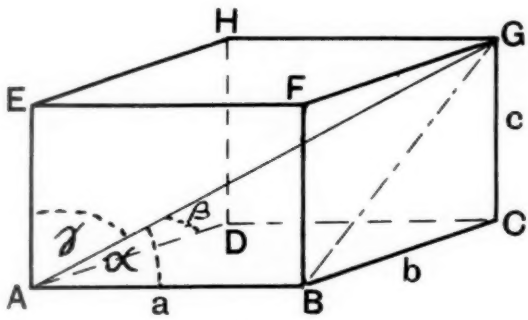


Fig. 3

To make the problem concrete let us take a, b, c of dimensions 6, 5, 4 units respectively. Then

(1) length of the projection of

$$AB=6\sqrt{\frac{41}{77}}=4.378 \text{ and } \alpha=46°52'$$

(2) length of the projection of

$$BC=5\sqrt{\frac{52}{77}}=4.109 \text{ and } \beta=55°17'$$

(3) length of the projection of

$$AE=4\sqrt{\frac{61}{77}}=3.560 \text{ and } \gamma=62°53'$$

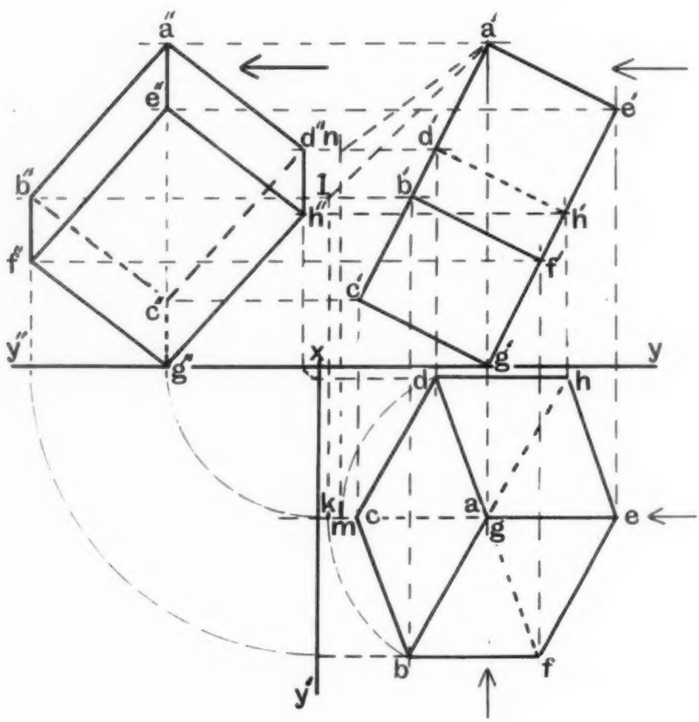


Fig. 4

We have when the block is standing with AG vertical now the lengths of the plans of the sides also the angles the sides make with the vertical. To make the geometry easier and still losing no gen-

erality we will suppose the side AE of length 4 units is parallel to the vertical plane, so that its elevation will be of true length. We therefore begin the elevation by drawing the upright line $a'g'$, Fig. 4, of length equal to $\sqrt{77}=8.775$ units and the line $a'e'$ inclined to it at $62°\ 53'$ and of length 4.000 units. Select a point for the plan of A. It is also g the plan of G. Draw the line cae through a parallel to xy and make gc and ae each equal to 3.560 units. On ac draw a triangle adc whose sides ad, cd are respectively 4.108 and 4.378 units respectively. Complete the parallelogram $adcb$. It is the plan of the 6×5 face that is seen. On ge $(=ac)$ as diagonal draw a similar parallelogram $ehfg$. It is the plan of the other 6×5 face that is not visible from above. Join bf, dh. These with ae are the plans of the three visible edges of length 4; gc, gf, gh are the three invisible edges of lengths 4, 5, 6; they must remain dotted.

To complete the elevation project upwards from all points in the plan. Draw $g'c'$ parallel and equal to $a'e'$ to meet the projection from c in c'. Its length is 4 units, of course. Join $a'c'$, $e'g'$. These cut the projections from d, b and f, h in d', b', and f', h' respectively. Join $b'f'$ by a full line and $d'h'$ by a dotted line.

That b', d', lie on $a'c'$ and f', h' lie on $e'g'$ follows from the fact that since the edge AE is parallel to the vertical plane the planes $ABCD$, $EFGH$ are perpendicular to the vertical plane and therefore appear strictly on edge, i.e. as lines. If you are in doubt of this find b' by the following method. Use the conception of a halfcone of semivertical angle $46°\ 52'$ and of slant length 6 cm; ab is the plan of one slant edge AB and $a'b'$ is the elevation, b' to be found. With a as center revolve ab around to cut eac in k. Draw $a'l$ inclined at $46°\ 52'$ to the vertical to meet the projection up from k in l. The length of $a'l$ is, naturally, 6 units. Draw a horizontal from l to meet the upward projection from b in b'. You will find that b' lies on $a'c'$. Similarly, as shown in the drawing, d' also lies on $a'c'$. Finally draw $b'f'$ as a full line and $d'h'$ as a dotted line. The front elevation is now complete.

The construction of a side elevation seen looking from the right as indicated by the left-pointing arrows is also shown. The method follows that used with the cube and needs no further description. It shows all the faces.

According to the Office of Education, 58.1% of the funds for public education are obtained through local property taxes. State taxes on incomes, sales, and other measures of business activity produce 37.4% of the funds for public schools. The Federal Government pays the remaining 4.5%.

A LABORATORY METHOD OF TEACHING PHYSICS

PAUL WESTMEYER
Instructor in Education, University of Illinois, Urbana, Ill.

If a teacher is fortunate enough to teach physics in a school where the laboratory is pretty well equipped he has a wonderful motivating tool within his grasp. Too often the laboratory work that is done in high school physics classes is either demonstration by the teacher or group work by the students following pre-arranged directions or, worse yet, a printed lab manual. Laboratory work can be so challenging to students and teacher, and consequently so rewarding, that it is a shame to waste the facilities and time with static, wornout procedures. What satisfaction can there be for a student to fill out blanks in a lab manual or for a teacher to grade same? Is there really much learning in this procedure? Granted that lab manuals *can* be used in ways that make them of real value, how often *are* they so used?

It is surprising to many to find that students in physics classes, who are usually juniors and seniors, can act in pretty mature ways if they are led to do so and are given the opportunity. They really can direct their own learning procedures. Of course, the teacher is there to teach the subject but if the students can teach themselves they learn so much more. How many times haven't we teachers observed that while students might not listen very well when we were explaining a principle they would listen raptly when the same principle was being explained by a fellow student?

A method I have used in two physics classes which intellectually were on opposite ends of the scale (I don't mean to insult the one group, they weren't "dummies" but neither were they far above average in intelligence while the other group was) in which the students have been encouraged to make rather free use of the lab has proven very successful both as to results academically and as to motivation. So far the method has been applied in the teaching of electricity and it has also been adapted for use in a chemistry class. It is as follows:

There is first some general class discussion on the subject, electricity, and then the class and teacher cooperatively set up a list of topics covering all aspects of the subject that the class in general wants to cover. This could probably just as well be set up by the teacher alone, for it isn't *what* we teach that counts as much as *how* we teach it, except that something is gained by way of motivation if the class has a hand in setting up the outline of material to be studied.

The second step is to set up a list of "laboratory investigations" to be made by the students. This list can be prepared in several ways —the students might set it up after scanning several textbooks and lab manuals, the teacher might make out the list himself, or the teacher and students can set up the list in a discussion session. No one should be bound to do only what is on this list nor should he be required to do all of it. The list also should not contain any directions for making the investigations; part of the purpose of the method is to get the students to develop their own initiative in this respect. Since we still cling to the beliefs that some areas of the study of physics absolutely *must* be covered in a high school course (and this may be right, the purpose here is not to argue this point but to present a method of teaching), some of the items on this list may be labelled required while others are optional.

After the preparation of these two lists the class is ready to begin its study of the subject. The members decide upon various topics which each will present to the class individually or in groups; perhaps some of the more difficult topics on the list are taken by the teacher but this is to be avoided if possible because the teacher is always handy to offer help if it is needed by any student in preparing his presentation. The idea is to cover the topics and do the laboratory investigations at the same time so as to be able to relate the reports to lab work as much as possible. To do this a schedule is set up, starting with perhaps three reports and one lab day a week, the other day being reserved for the weekly test which is a way of continual evaluation of the progress of the work. Later when the topics become more difficult a "catch-up day" is arranged perhaps once a week for those who are having trouble to talk over the problems in a group with one another and the teacher. This time is spent in the lab by those who do not need the extra help; they are expected to do more of the optional investigations.

When a report is being presented the students are not required to attend. Instead if they think that they are thoroughly familiar with the subject, having read material on it in reference books which are always available, they may elect to work in the laboratory. Obviously this splits the teacher's time between the report session and the lab and necessitates a good bit of trust placed in the students. Probably the students will give better reports and have freer discussion if the teacher is not present all of the time; however, there are times when the teacher should be there to clarify problems that the reporter can't handle. The teacher will have to judge each day where he needs to spend most of his time.

The students working in the laboratory during the reports cannot

be under the close supervision of the teacher and many administrators might balk at this because those expensive instruments are not to be touched by students, the teacher is supposed to demonstrate with them. The students will never learn how to use an instrument by watching someone else use it, they must handle it themselves. Anyway, the mortality isn't going to be very great if proper methods of acquainting students with the potentialities and limitations of each instrument are used. This can be done by mimeographing some facts concerning each instrument that is to be used. Also the students can help one another to learn proper techniques if they are encouraged to do so, and the teacher is there a part of the time to oversee and suggest proper techniques. If a large proportion of the class will be going on to college, and perhaps in other cases too, the instrument-handling situation can be further helped by giving a weekly "technique grade" based on the way in which the student handles himself and the instruments in the lab.

Instead of the usual formal reports of lab work each pupil keeps a notebook, which is more scientific anyway, in which he records (1) notes taken from the discussions, (2) any reading notes that he thinks might be helpful (the book does receive a grade but its chief purpose is to help the student better to understand and remember facts and principles for the sake of tests that must be given), and (3) a description of just what was done in performing a laboratory investigation, what data were observed, and what the implications of this are. The description is necessary since there are no formal printed directions to follow; the students use their own ingenuity and knowledge in performing these investigations. They do this in groups of two to four, which groups are not set by the teacher but are formed on the spur of the moment by those who need to work on the same subject.

The study ends when all of the topics and investigations on the orginal lists have been covered by the class. It would logically terminate with a major test covering the material. However, with such a method it is to be hoped that some of the students would be led into further study of certain phases of the subject. Thus the method could lead to the very desirable situation of having several students doing individual work in areas of particular interest to them. In any case the method of learning by developing one's own procedures, in cooperation with others, and then carrying out these procedures to successful conclusions should be both useful and very rewarding at the time of action, and later on I hope.

A laugh is worth a hundred groans in any market.—Lamb

EDUCATING THE GIFTED PUPIL IN MATHEMATICS AND SCIENCE

AN IMPORTANT PROJECT FOR OUR NATION

MONTE S. NORTON
Whittier Junior High School, Lincoln, Nebr.

American public schools are dedicated to the principle of providing opportunities for the optimum development of each pupil. In harmony with this obligation and the objectives of education in a democracy is the important task of identifying and making provisions for the student with exceptional ability in the fields of mathematics and science. Schools must not expect their future leaders in mathematics and science to stumble by chance into their proper place in society.

> The fruits of genius are seldom the result of pure inspiration or happy accident, but rather the outcome of prolonged thinking and in consequence of rigorous specialized training—creative genius does not arise in a vacuum.[1]

The gifted pupils of today must be identified and expertly guided early in life if they are to become the leaders of tomorrow. The education of the gifted pupil must be planned with the keenest and most intelligent insight. The nation needs individuals with a well-developed sense of values along with high intellectual qualities. Such capable people need guidance in order to appreciate their abilities and choose appropriate goals. Maximum use of highly talented individuals is requisite in meeting our society's increasingly complex problems, and is necessary if these individuals are to enjoy maximum personal achievement and contribute to the progress and human welfare of the nation. "That society is wise which allows the new ideas and creative work of genius to find expression."[2]

The technological advances of the world have been made possible by the increasing number of specialists. Reports indicate that the United States has doubled its supply of technical specialized personnel during the last decade. In the advanced technological state in which we live, it is impossible to be a successful engineer, economist, physician, and the like unless one has a thorough background in the basic sciences. Since mathematics is basic to science, this field is essential, then, for continued advances in engineering and other sciences.

> Science, industry, business, and government are desperately looking for individuals of high intelligence and sound training to occupy positions of leader-

[1] Gertrude Howell Hildreth, *Educating Gifted Children*, Harper and Brothers, New York, 1952, pp. 5 and 52.

[2] Educational Policies Commission, *Education of the Gifted*, National Education Association of the United States and the American Assoc. of School Administrators, June, 1950, p. 12.

ship. . . . There are many capable individuals who are not being prepared for the great work they can do. We must find them and save them for themselves and for society.[3]

Paul A. Witty points out that, ". . . full utilization of the best ability of the nation is essential for continued leadership and progress."[4]

But we need the abilities of our brightest persons for more than material progress. We are in a struggle to determine by which goals and ideals the people of the world will live. We believe that democracy and freedom offer the best answers for man today. In our effort to remove slavery and darkness from the world and in our attempt to help all people learn to live in amity and peace, we need spiritual guidance and courageous leadership—talented men and women equipped through education to find new solutions to old problems. We need brilliance in diplomacy and in human relations. We need the resourcefulness and the imagination of the gifted to create a better world. . . . The question is not: "Can we afford a better program for gifted children?" but rather: "Can we afford *not* to develop these human resources to the utmost?"[5]

There is a potential source of scientists and mathematicians that needs to be tapped. Only 40% of the high school graduates of college ability are granted a college degree. What happens to the other 60%? What happens to the large pool containing many potential scientists and engineers? Twenty per cent drop out during college and 40% never enter.[6]

According to reports, many high school graduates cannot afford the costs of a college eduction but many drop out because of a failure to appreciate the importance of college studies and the contributions they could possibly make to society. Proper instruction and guidance in the public schools may be important factors in the solution of this problem. Special effort should be made to provide better opportunities for the rapid learner in the high school, for it is felt that proper guidance and instruction can prepare the capable student for leadership in the area in which he can be of most value to himself and society. All pupils, including the gifted, should spend a major portion of their school life on the general studies designed to develop the common needs of all—studies that will help the individual achieve and become proficient in the general objectives of education. Beyond this general education suggested for all students, the gifted pupils should have additional experiences appropriate to their special needs. It is felt by this writer that an awareness should be developed of the waste of human resources, and a program initiated which will provide maximum opportunity for the youth of our nation.

To capitalize on the rich resources of human talent which gifted children and youth possess, the schools and colleges must give special attention to the education of their gifted students.[7]

[3] D. A. Worcester, *The Education of Children of Above-Average Mentality*, University of Nebraska Press, 1955 pp. 3 and 4.

[4] Paul H. Witty, "Today's Schools Can Do Much for the Gifted Child," *The Nations Schools*, Arthur H. Rice, Editor, Los Angeles, California, 1956, Vol. 57, No. 2, p. 66.

[5] *Ibid.*, p. 65.

[6] U. S. Department of Health, Education, and Welfare, *Education for the Talented in Mathematics and Science*,, Bulletin 1952, No. 15, Washington, D. C., p. 3.

[7] Educational Policies Commission, *op. cit.*, p. iii.

The maximum welfare for a group is achieved when each member of the group contributes as much as he is able. . . . The democratic ideal can be most fully attained when every individual has opportunity for educational experience commensurate with his abilities and for vocation responsibility commensurate with his qualifications.[8]

The future of our country and of democracy as a way of life depend to a considerable degree upon the widespread recognition and development of our greatest resource—gifted children and youth.[9]

The foregoing facts indicate that the public schools *must* make special provisions for the gifted pupil. In making provisions for the gifted pupil in mathematics and science, teachers need books, and materials for supplementary study; time and facilities for preparing instructional material; time for student counseling and individual instruction; and equipment for developing meaningful learning activities. This type of program needs the full support of administrators and the community in general. M. H. Ahrendt points out that, " . . . regardless of the fine equipment that a teacher may have for dealing with students who show aptitude in mathematics, he is almost helpless without the assistance of his administrators."[10] American society needs the most capable people as its leaders. Public schools and other educational institutions can do much to meet this need by providing the best possible programs for the optimum development of all talented youth. Educational institutions, without the interest and assistance of the American people are almost helpless. To provide the education that talented pupils should have, financial assistance as well as a genuine appreciation for developing each individual to a maximum is needed. Educators, parents, and all other Americans should develop a keen awareness of the present problems that now exist in the education of the gifted pupil. This is indeed an important project for our nation.

---

[8] *Ibid.*, p. 2.

[9] Paul A. Witty, *The Nations Schools, op. cit.*, p. 72.

[10] M. H. Ahrendt, "Education of the Mathematically Gifted," *Phi Delta Kappan*, Vol. XXIV, No. 7, April, 1953, pp. 285–7.

RARE TRUMPETER SWANS ARE DYED YELLOW IN OREGON

There is no need to change your way of living if you spot a yellow swan. For there is no relationship between the unusual yellow swan and "pink elephants."

Out on the Malheur Bird Refuge in eastern Oregon the U. S. Fish and Wildlife Service has dyed six trumpeter swans yellow so their dispersion pattern may be traced if they leave the Malheur Waterfowl Refuge.

The trumpeter, biggest of our North American waterfowl, was near extinction 20 years ago. At that time only 73 of the big birds could be counted. With careful watching, the trumpeter flock has grown to over 600. The yellow dye is part of the control program. Observers have reported the low-flying swans, with their six-foot wing spread, migrating between nesting areas in the United States and Canada.

PROBLEM DEPARTMENT

CONDUCTED BY MARGARET F. WILLERDING
San Diego State College, San Diego, Calif.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution or proposed problem sent the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The editor of the Department desires to serve his readers by making it interesting and helpful to them. Address suggestions and problems to Margaret F. Willerding, San Diego State College, San Diego, Calif.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Solutions should be in typed form, double spaced.

2. Drawings in India ink should be on a separate page from the solution.

3. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

4. In general when several solutions are correct, the one submitted in the best form will be used.

LATE SOLUTIONS

2475. *Neil L. Norcross, Hartford, Mich.*

2515. *Proposed by Paul J. Malie, Hickory, Pa.*

Mr. K. always takes the 7:30 train home. His chauffeur arrives at the station just as the train is due every day. One day, Mr. K. took the 6:25, but his secretary forgot to call the chauffeur; accordingly, Mr. K. got off the train at the station, and seeing no chauffeur, started to walk to his home at the rate of 3 miles per hour. He met his chauffeur who turned and got him home 10 minutes earlier than usual. Assuming that no time was lost in car operation, what was the speed of the car?

Solution by Sister Mary Leona, Saginaw, Michigan

On the day that Mr. K. walked part of the way home, let d_w+d_r represent the distance from the station to his home, where d_w is the distance that he walked and d_r the distance that he drove. On that trip the chauffeur traveled $2d_w$ miles less and saved $\frac{1}{6}$ hour. If s is the speed of the car,

$$2d_w/s = \tfrac{1}{6}, \quad \text{or} \quad d_w/s = \tfrac{1}{12}$$

Since Mr. K. arrived at the station $1\frac{1}{12}$ hours earlier and arrived home $\frac{1}{6}$ hour earlier than usual, the time of the journey from the station to his home compared to the usual time taken is

$$d_w/3+d_r/s-(1\tfrac{1}{12}-\tfrac{1}{6})=d_w/s+d_r/s$$

Substituting from the above equations and simplifying, the rate of the car is

$$s=36 \text{ miles per hour}$$

Solutions were also offered by Julian H. Braun, San Diego, Calif.; J. Byers King, Denton, Md.; and Peter Landweber, Iowa City, Iowa.

2516. *Proposed by Howard D. Grossman, New York, N.Y.*

If from opposite vertices A and C of parallelogram $ABCD$ lines AO and CO are drawn meeting at O and making equal angles with AB and CB respectively, then angle AOB equals angle COD.

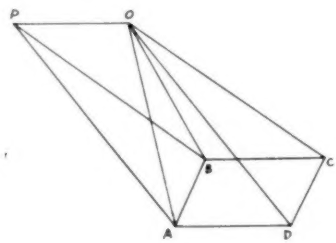
Solution by the proposer

Complete parallelogram $ADOP$.

Since PO, AD, and BC are all equal and parallel, BP is parallel to CO.

Since angle ABP equals angle DCO equals angle OAD equals angle AOP, points A, B, O and P lie on a circle.

Therefore angle AOB equals angle APB equals angle DOC.



2517. *Proposed by Dwight L. Foster, Tallahassee, Fla.*

If

$$a+\frac{bc-a^2}{a^2+b^2+c^2}$$

be not altered in value by interchanging a pair of the letters a, b, c not equal to each other, it will not be altered by interchanging any other pair, and it will vanish if $a+b+c=1$.

Solution by Alan Wayne, Baldwin, N. Y.

Designate the given expression by $f(a, b, c)$. Then, obviously, $f(a, b, c)=f(a, c, b)$. Suppose $a\neq b$, and $f(a, b, c)=f(c, b, a)$. Then $f(a, b, c)-f(c, b, a)=0$, whence $a^2+b^2+c^2=a+b+c$ and $f(a, b, c)=(ab+ac+bc)/(a^2+b^2+c^2)$. Since this last function is symmetric in a, b, and c, it will remain unchanged in value when any two of a, b, and c are interchanged. Also, if $a+b+c=1$, then $(a+b+c)^2=a^2+b^2+c^2$, whence $ab+ac+bc=0$, so that $f(a, b, c)$ vanishes.

A solution was also offered by the proposer.

2518. *Proposed by A. R. Haynes, Tacoma, Wash.*

If AC and BD are diagonals of the square $ABCD$ and M is any point on its circumcircle, show that the points in which MA and MC cut BD are conclyclic with the points in which MB and MD cut AC—the diagonals being produced as required.

Solution by the proposer

Let the intersection of the diagonals be taken as the origin with diagonal BD as the Y-axes and CA as the X-axis. Then we have

$$A(r, 0), \qquad B(0, r), \qquad C(-r, 0), \qquad D(0, -r).$$

Take M on the circumcircle in the 4th quadrant:

$$M(a, -b), \quad \text{then} \quad a^2+b^2=r^2 \tag{1}$$

Line DM cuts CA produced in P_1.

$$\begin{cases} D(0, -r) \\ M(a, -b) \end{cases} \qquad y+r=\frac{r-b}{a}x$$

Since

$$y=0 \quad \text{for} \quad P_1, \qquad x=\frac{ar}{r-b}: P_1\left(\frac{ar}{r-b}, 0\right) \tag{2}$$

Line BM also cuts CA in P_2.

$$\begin{cases} B(0, r) \\ M(a, -b) \end{cases} \qquad y-r=\frac{r+b}{-a}x$$

Also

$$y=0 \quad \text{for} \quad P_2, \qquad x=\frac{ar}{r+b}: P_2\left(\frac{ar}{r+b}, 0\right) \tag{3}$$

The centers of circles passing through P_1 and P_2 will lie on the $\perp$ bisector of this segment: where:

$$x=\frac{OP_1+OP_2}{2}=\frac{\frac{ar}{r-b}+\frac{ar}{r+b}}{2}=\frac{r^2}{a} \tag{4}$$

Similarly, the co-ordinates for P_3 and P_4 on the Y-axis are found to be

$$P_3\left(0, \frac{-br}{r+a}\right) \tag{5}$$

$$P_4\left(0, \frac{-br}{r-a}\right) \tag{6}$$

And the locus for the centers of circles passing through P_3 and P_4 is

$$y=-\frac{r^2}{b} \tag{7}$$

Now if P_1, P_2, P_3, P_4 are concylic the circle must have its center on the intersection of the locii (lines)

$$\begin{cases} x=\frac{r^2}{a} \\ y=-\frac{r^2}{b} \end{cases} \quad \text{or} \quad O_p\left(\frac{r^2}{a}, -\frac{r^2}{b}\right) \tag{8}$$

For such a circle the radius $O_p P_2$ must be the same as the radius O_pP_3. Now

$$O_pP_2\left(\frac{ar}{r+b}, 0\right)=\sqrt{\left(\frac{r^2}{a}-\frac{ar}{r+b}\right)^2+\left(\frac{r^2}{b}\right)^2}$$

(Substituting $r^2=a^2+b^2$ as required)

$$=r\sqrt{\left(\frac{b}{a}\right)^2+\left(\frac{r}{b}\right)^2}$$

$$=\frac{r}{ab}\sqrt{b^4+a^2b^2+a^4}$$

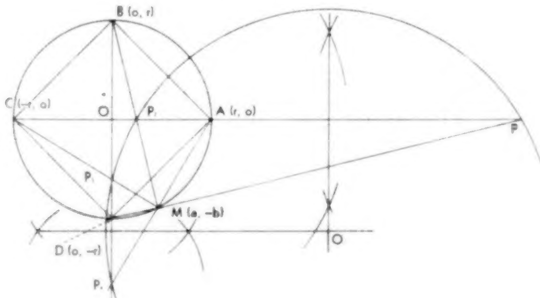
and

$$O_pP_3\left(0, \frac{-br}{r+a}\right)=\sqrt{\left(\frac{r^2}{a}\right)^2+\left(-\frac{r^2}{b}+\frac{br}{r+a}\right)^2} \tag{9}$$

$$=r\sqrt{\left(\frac{r}{a}\right)^2+\left(\frac{a}{b}\right)^2}$$

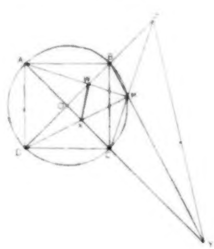
$$=\frac{r}{ab}\sqrt{b^4+a^2b^2+a^4} \tag{10}$$

Hence the circle with center at O_p which passes through P_1 and P_2 with radius (9) also passes through P_3 and P_4 by (10) and the 4 points are concyclic.



2518. *Second Solution by Peter Landweber, Iowa City, Iowa.*

Prove points W, X, Y, and Z concyclic.



$$\measuredangle BMD \text{ and } AMC \text{ are rt.} \measuredangle$$

$$\triangle ODX \sim \triangle BDM.$$

AC and BD are $\perp$ bisectors to one another.

$$\triangle OYB \sim \triangle BDM$$

$$\therefore\ \triangle ODX \sim \triangle OYB$$

$$\therefore\ \frac{OY}{OD}=\frac{OB}{OX}$$

$$\triangle OAW = \triangle CAM$$

$$\triangle OZC \sim \triangle CAM$$

$$\therefore\ \triangle OZC \sim \triangle OAW$$

$$\therefore\ \frac{OZ}{OA}=\frac{OC}{OW}$$

$$OY\cdot OX=OB\cdot OD=\overline{OB}^2$$

$$OZ\cdot OW=OA\cdot OC=\overline{OA}^2$$

$$\therefore\ OY\cdot OX=OZ\cdot OW, \text{ and } \frac{OY}{OZ}=\frac{OX}{OW}$$

$\therefore$ right $\triangle WOX \sim$ rt. $\triangle YOZ$

$$\therefore\ \angle OWX=\angle OYZ$$

$$\angle OWX+\angle ZWX=180°$$

$$\therefore\ \angle OYZ+\angle ZWX=180°$$

$\therefore$ Quadrilateral $WXYZ$ may have a circle circumscribed about it, and

$\therefore$ Points W, X, Y, and Z are concyclic.

2519. *Proposed by Hugo Brandt, Chicago, Ill.*

A cylindrical hole with radius r is drilled along a diagonal of a cube of side s. Find the volume of the material removed.

Solution by Sister Mary Leona, Saginaw, Michigan

The form of the removed material consists of ends that are regular triangular pyramids with base-radius r, a central portion which is a right circular cylinder connected to each end-pyramid through a transition piece with one base equal to that of the pyramid and the other base a circle with radius r.

Let $VRV'S$ be a section through diagonally opposite edges of the cube along the drilled hole (Fig. 1). Figure 2 is an enlarged detail of one vertex.

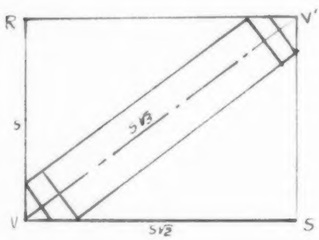


FIG. 1

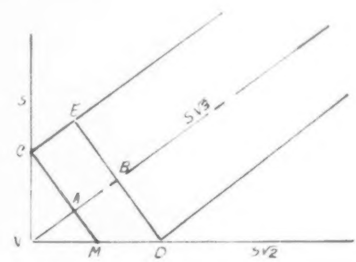


FIG. 2

1. The radius CA of the circle circumscribing the base of the pyramid is r, therefore the area B of the base is

$$B=\frac{3\sqrt{3}r^2}{4}$$

The following proportions are derived from the dimensions of the cube:

$$CV:CA:VA=\sqrt{3}:\sqrt{2}:1$$

Since $CA=r$, the altitude

$$VA=\frac{r}{\sqrt{2}},$$

and the volume of the pyramid V_p is

$$V_p=\frac{\sqrt{6}}{8}\,r^3$$

2. The altitude of the cylinder is the diagonal of the cube less $2VB$. Since $BD=r$, the value of VB can be easily derived from the proportions

$$VB:VD:BD=\sqrt{2}:\sqrt{3}:1.$$

$VB=r\sqrt{2}$. Therefore, the volume of the cylinder is

$$V_c=(s\sqrt{3}-2\sqrt{2}r)\pi r^2.$$

3. Section $CMDE$ of Fig. 2 is the remaining portion of the required volume. The base CM is an equilateral triangle, the same as the base of the pyrmaid, and ED is a circle of radius r. The altitude of this part is

$$AB=VB=VA=\frac{r}{\sqrt{2}}$$

The area of any section is

$$A=\pi r^2h+\frac{3r^2}{2}\sin\phi,$$

where h varies from 0 to 1 measured from the triangular base, and

$$\phi=(1-h)\frac{2\pi}{3}$$

ϕ is the central angle subtended by one of the three equal chords in the boundary of any right section.

Replacing ϕ and separating the variable

$$A=\pi r^2h+\frac{3r^2}{2}\sin(1-h)\frac{2\pi}{3}$$

$$A=\pi r^2h+\frac{3r^2}{2}\left(\sin\frac{2\pi}{3}\cos\frac{2\pi}{3}h-\cos\frac{2\pi}{3}\sin\frac{2\pi}{3}h\right)$$

$$A=\pi r^2h+\frac{3r^2}{2}\left(\frac{\sqrt{3}}{2}\cos\frac{2\pi}{3}h+\frac{1}{2}\sin\frac{2\pi}{3}h\right)$$

The mid-section, where $h=\frac{1}{2}$, is the average of all right sections. At this value of h

$$A=\frac{\pi r^2}{2}+\frac{3\sqrt{3}r^2}{4}$$

With altitude equal to $r/\sqrt{2}$, the volume V_s of this figure becomes

$$V_s=\left(\frac{\pi r^2}{2}+\frac{3\sqrt{3}r^2}{4}\right)\frac{r}{\sqrt{2}}$$

or

$$V_s=\left(\frac{\pi}{2\sqrt{2}}+\frac{3\sqrt{3}}{4\sqrt{2}}\right)r^3$$

The total volume of the removed material

$$V=2V_p+2V_s+V_c$$

$$V=\frac{\sqrt{6}r^3}{4}+\left(\frac{\pi}{\sqrt{2}}+\frac{3\sqrt{3}}{2\sqrt{2}}\right)r^3+(s\sqrt{3}+2\sqrt{2}r)\pi r^2$$

Simplifying, the volume of the material is

$$V=(\sqrt{6}+5\pi\sqrt{2})r^3+\pi\sqrt{3}sr^2$$

A solution was also offered by the proposer.

2520. *Proposed by N. Kailasamaiyer, Madras, India.*

Resolve into factors

$$a(1-b^2)(1-c^2)+b(1-c^2)(1-a^2)+c(1-a^2)(1-b^2)-4abc$$

Solution by J. Byers King, Denton, Md.

Taking $(1-b^2)$ out of the first and third terms

$$(1-b^2)(a-ac^2+c-a^2c)+b(1-c^2-a^2+a^2c^2-4ac)$$

Forming a quadratic in b by rearranging

$$-b^2[(a+c)-ac(a+c)]+b(1-2ac+a^2c^2-a^2-2ac-c^2)+1[(a+c)-ac(a+c)]$$

$$-b^2(a+c)(1-ac)+b[(1-ac)^2-(a+c)^2]+(a+c)(1-ac)$$

Factoring

$$[b(a+c)-(1-ac)][-b(1-ac)-(a+c)]$$

or

$$(ab+bc-1+ac)(abc-b-a-c)$$

A solution was also offered by the proposer.

STUDENT HONOR ROLL

The Editor will be very happy to make special mention of classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

Editor's Note: For a time each student contributor will receive a copy of the magazine in which his name appears.

The Student Honor Roll for this issue appears below.

2515. *Larry Kramm and Richard Grogan, Woodrow Wilson High School, Long Beach, Calif.*

PROBLEMS FOR SOLUTION

2539. *Proposed by Vincent C. Harris, San Diego State College.*

Which requires less material (that is, has less surface area), a cylinder of the most economical proportions or a cube, if they contain the same volume?

2540. *Proposed by James Dowdy, San Antonio, Texas.*

Prove that

$$\pi=2i\log\frac{1-i}{1+i},$$

where $i=\sqrt{-1}$.

This was first proved by Count Fagnano (1672-1766).

2541. *Proposed by Brother Felix John, Philadelphia, Pa.*

Is the following expression an identity or an equation? Prove or solve it, if possible.

$$\frac{\sin X+\sin 2X}{\sin X+\sin 3X}=\frac{\sin 3X(1-\tan^2 X)}{\sin 4X(2\cos X-1)}$$

2542. *Proposed by Nathan Altshiller-Court, University of Oklahoma.*

Find the length (in terms of the sides) of the segment determined on a side of a triangle by the bisectors of the angles formed by the median relative to that side and the parallel to that side through the opposite vertex.

2543. *Proposed by Hugo Brandt, Chicago, Ill.*

Prove

$$|a_1\ b_2\ c_3|^2 = |A_1\ B_2\ C_3|$$

where

$$|a_1\ b_2\ c_3|$$

stands for

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

and A_1, etc. is the minor of a_1, etc., or

$$A_1 = \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$$

2544. *Proposed by Julius Sumner Miller, El Camino, Calif.*

A rough sphere of radius r rolls inside of a fixed hollow sphere of radius $4r$. Find the least velocity in its lowest position so that it may retain contact with the highest point of the hollow sphere.

BOOKS AND PAMPHLETS RECEIVED

SCIENCE IN TODAY'S WORLD, by Maurice U. Ames, *Principal, Frank D. Whalen Junior High School, New York City;* Arthur O. Baker, *Directing Supervisor of Science, Cleveland Board of Education;* and Joseph F. Leahy, *Science Instructor, Herrick Junior High School, Cleveland, Ohio.* Cloth. 280 pages. 17×23 cm. 1956. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. Price $3.32.

ARITHMETIC FOR ENGINEERS, Fifth Edition, by Charles B. Clapham, *Author of Metric System for Engineers.* Cloth. Pages xiii+540. 13.5×21.5 cm. 1956. John F. Rider, Publisher, Inc., 480 Canal Street, New York 13, N. Y. Price $6.50.

ABACS OR NOMOGRAMS, by A. Giet, *Ecole Nationale des Ingénieurs Arts et Métiers, Paris.* Cloth. Pages ix+225. 13.5×21.5 cm. Philosophical Library, 15 East 40th Street, New York 16, N.Y. Price $12.00.

THE GENERATION OF ELECTRICITY BY WIND POWER, by E. W. Golding. Cloth. Pages xvi+318. 14×22 cm. 1955. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $12.00.

ELECTRICAL INTERFERENCE, by A. P. Hale, Grad. I.E.E. Cloth. Pages vii+122. 12×18.5 cm. 1956. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $4.75.

MEASUREMENTS OF MIND AND MATTER, by G. W. Scott Blair, *National Institute for Research in Dairying, University of Reading.* Cloth. 116 pages. 13.5×21.5 cm. 1956. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $4.50.

CRYPTANALYSIS, A STUDY OF CIPHERS AND THEIR SOLUTION, by Helen Fouche Gaines. Paper. Pages vi+237. 14×21.5 cm. 1956. Dover Publications, Inc., 920 Broadway, New York 10, N. Y. Price $1.95.

INVERSE FEEDBACK, Edited by Alexander Schure, Ph D., Ed.D. Paper.

Pages vii+48. 14×21.5 cm. 1956. John F. Rider, Publisher, Inc., 480 Canal Street, New York 13, N. Y. Price 90 cents.

BASICS OF PHOTOTUBES AND PHOTOCELLS, by David Mark. Paper. Pages vii+128. 14×21.5 cm. 1956. John F. Rider, Publisher, Inc., 480 Canal Street, New York 13, N. Y. Price $2.90.

TV MANUFACTURERS' RECEIVER TROUBLE CURES, Volume 8, Edited by Milton S. Snitzer. Paper. Pages x+117. 14×21.5 cm. 1956. John F. Rider, Publisher, Inc., 480 Canal Street, New York 13, N. Y. Price $1.80.

JOLLY NUMBERS, Book One, Revised Edition, by Guy T. Buswell, University of California, Berkeley; William A. Brownell, *University of California, Berkeley;* and Lenore John, *Laboratory Schools, The University of Chicago, Chicago, Illinois.* Paper. 74 pages. 22×29.5 cm. 1956. Ginn and Company, Statler Building, Boston 17, Mass. Price 68 cents.

JOLLY NUMBERS, Book Two, Revised Edition, by Guy T. Buswell, *University of California, Berkeley;* William A. Brownell, *University of California, Berkeley;* and Lenore John, *Laboratory Schools, The University of Chicago, Chicago, Illinois.* Paper. Pages iv+77. 21.5×29.5 cm. 1956. Ginn and Company, Statler Building, Boston 17, Mass. Price 88 cents.

JOLLY NUMBERS, Primer, Revised Edition, by Guy T. Buswell, *University of California, Berkeley;* William A. Brownell, *University of California, Berkeley;* and Lenore John, *The University of Chicago, Chicago, Illinois.* Paper. 65 pages. 22.5×29.5 cm. 1956. Ginn and Company, Statler Building, Boston 17, Mass. Price 64 cents.

HOW TO BECOME A RADIO AMATEUR, Fifteenth Edition, by the Headquarters Staff of the American Radio Relay League. Paper. 148 pages. 16×24 cm. 1956. American Radio Relay League, West Hartford 7, Conn. Price 50 cents.

TEACHERS OF CHILDREN WHO ARE DEAF, Prepared by Romaine P. Mackie and Others. Bulletin 1955, No. 6. Pages viii+87. 15×23 cm. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price 35 cents.

TOMORROW'S SCIENTISTS AND ENGINEERS, Prepared by the Research Committee and the Education Department of the National Association of Manufacturers. Paper. 15 pages. 15×21.5 cm. Education Department, National Association of Manufacturers, 2 East 48th Street, New York 17, N. Y.

BOOK REVIEWS

THE WORLD OF ATOMS, AN INTRODUCTION TO PHYSICAL SCIENCE, by J. J. G. McCue, *Formerly Associate professor of Physics, Smith College;* assisted by Kenneth W. Sherk, *Professor of Chemistry, also of Smith College.* Cloth. xi+659 pages. 16.0×23.5 cm. 1956. The Roland Press Company, 15 East 26th Street, New York 10, N. Y. Price $6.50.

Can the varied aspects of physical science be brought into a unity without producing a structure that seems to have a metastable equilibrium? Is it possible to find a single thread that, running through, will give the whole a satisfying sense of oneness? Dr. McCue proposes to so produce in *The World of Atoms.*

In a first section headed, "Mechanics" he traces the evolution of the Newtonian system as the answer in history to the astronomers' puzzlement over our solar system. A second section, "Heat" traces the use of the theory of discontinuous matter for explanation of gas properties and then to its more general application to energy in all matter. A third section, "Foundations of

Chemistry," initiates the other segment of physical science. In this section atomism is first presented. Following that "Some Chemical Reactions" supply information in preparation for a "tie-in" with the physics of the first two sections.

Sections on "Electricity" and "Solutions" serve well to bring both physics and chemistry together for their exposition. The empirical "Classification of the Elements" raises questions that find answers in "The Structure of Atoms". Architecture of atoms makes possible a more explicit attention to "Chemical Bonding." And, as evidence that *The World of Atoms* is not yet fully explored, a last section on "The Atomic Nucleus."

Each section is subdivided into from three to ten chapters, each of a content that serves as a one day assignment. These chapters are serviced by questions of both recall and application. Many of the chapters are followed by "Suggestions for Further Reading." There are twenty-four full page portraits of persons who were contributors to the evolution of atomism. Some two hundred or more other illustrations and diagrams with helpful legends minister to fuller understanding.

The author's preface promise is "that an introductory course should teach how a scientist goes about his work. . . . The student (should) be shown how the scientist arrived at (his) beliefs. . . . So far as possible, the argument should be an operational one; each concept resting on an experiment." Material and treatment in this book bear out the plan and intent proposed. Even though "written for those who are not studying to be scientists" there is little evidence of a "playing down to their level" in either vocabulary or symbolism. Those of lower academic skill are not always going to find it readily readable. The author recognizes that there are capable students who may not be majoring in sciences.

Here is a contribution to the teaching of physical science that should be read by, not only the teachers of the general courses, but by every one who teaches either chemistry or physics. With a view to adoption? Maybe; but more importantly with a view to his better understanding.

B. CLIFFORD HENDRICKS
Longview, Washington

AN INTRODUCTION TO THE ELECTRONIC THEORY OF VALENCY, Third Edition, by J. C. Speakman, M.Sc., Ph.D., D.Sc., *Senior Lecturer in Chemistry in the University of Glasgow.* Cloth. Pages vii+180. 13.0×19.0 cm. 1955. Edward Arnold (Publishers) Ltd., London. $2.50.

This was first issued in 1935. The "more drastic revision" here offered organizes its contents in terms of potential service for chemists. In Part One is offered a sort of "that-without-which" a modern "intending chemist" can not work. It consists of four chapters: Data and Principles; Electrovalency; Covalency and Dative Valency.

Part Two, using four fifths of the page space, is devoted to the "application of more subsequent theoretical developments to actual chemical substances." Here are found chapters on: Wave Mechanical Interpretation of Valency; Metrical Aspects of Valency; Electro Related to Covalency; Other Types of Interatomic and Intermolecular Forces; Valency of the Long Periods; Acids and Bases; and Quantivalence, the Molecule and the Bond.

A thirteen item bibliography, a table of "physical constants and units," an author and a subject index are provided. Eight figures and thirteen tables are a part of the textual material.

The treatment is historical in its orientation with a creditably successful presentation that keeps understandably and readably within the ken of the non-mathematical reader. For those who have "bogged down" in an attempt to follow Sedgwick's *The Electronic Theory of Valency* or Pauling's *The Nature of the Chemical Bond* this book may be a welcome help. It is certainly a more recent and up-to-date issue.

B. CLIFFORD HENDRICKS

LOGIC AND SCIENTIFIC METHODS: AN INTRODUCTORY COURSE, *Second Edition*, by Herbert L. Searles, *University of Southern California*, Cloth. Pages viii +378. 14×21.5 cm. 1956. The Ronald Press Company, 15 East 26th St., New York 10, N. Y. Price $4.25.

This is the second edition of a text that first appeared in 1948 and enjoyed considerable popularity. The author is of course well-known in philosophical circles and has contributed many papers and reports in his area.

The book attempts to illustrate the development of systems of logic by beginning with certain examples of logical reasoning familiar to nearly all students and making a transition to the more abstract aspects of this field of study.

The book is divided into three major sections, the first, *Logic and Meaning* which is descriptive in nature; the second, *Deductive Logic*, which deals essentially with formal systems of deductive logic; and the third, *Scientific Methods*, which deals with inductive methodology, cause and effects relationships, statistical methods, and the fallacies of hypotheses. About equal space is devoted to the second and third sections.

The reviewer has read the book carefully throughout and reread a number of different sections. There are several impressions one gets from such reading. The first is that the book is scholarly and complete. The second is that it is not easy reading. While it may be used in an introductory course in logic, or in philosophy, it is likely that a student would profit most if he had some understanding of the nature of science and the logic of reasoning prior to using this book.

There are two questions the reviewer has. Why is there such a sharp dichotomy, at least in the organization of the book, between deductive logic and scientific method? Are they so mutually exclusive? True, certain relationships between the two are indicated, but the dichotomy does seem to be drawn rather sharply. Also, why the brevity of the treatment of statistics? Not enough to take a bath, but just enough to make one uncomfortably damp.

The reviewer realizes, however, the tremendous breadth of the field to be covered, and congratulates the author for having done so well. Nevertheless, as with other "introductory books" it may be wondered why certain areas are not left out and the space devoted to doing others more thoroughly.

It is a good textbook, and probably a good reference, for a scientist or mathematician to have.

GEORGE G. MALLINSON

THE LITTLE GIANT OF SCHENECTADY. A STORY OF CHARLES STEINMETZ, by Dorothy Markey. Cloth. 191 pages. 13×20.5 cm. 1956. Aladdin Books, New York. Price $1.75.

Here is a little book for the delight of everyone who reads, the life of Charles Steinmetz. It is a very small book to tell the story of so great a genius, but by the time you have finished the less than 200 pages you will feel that you are a personal acquaintance and friend of the great inventor. Starting with life in Switzerland, where he had gone in exile after his short article against the Prussian military had been published in his home city of Breslau, he was easily persuaded to go to America. It was June 1, 1889 when he arrived with his American friend, Oscar Asmussen. Every sentence is a new and startling thought. Carl, the German, becomes Charles, the American. His first job was work on the blue prints for an electric streetcar motor. He was now in his own field. The remainder of the book tells of one success after another interspersed with minor setbacks. He became chief research engineer of a great electric company. His care of all animals and rare plants, his delight in having children come in to see them, his great work on alternating currents, his camp on Viele Creek, all are told in few but startling words. At Viele Creek he learned to know Roy Hayden, and later became "Dad" to Roy and his wife Corinne. You will enjoy every word of it and will understand why he is known as "The Little Giant of Schenectady."

G. W. W.

THE ELEMENTS OF CHROMATOGRAPHY (With eleven colored plates and twenty-seven other figures.), by Trevor Illtyd Williams, M.A., B.Sc., D.Phil., F.R.I.C. Cloth. Pages iv+90. 12.7×18.4 cm. 1953. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. $3.75.

This is a completely rewritten issue of a publication first offered in 1946. It includes especial attention to recent advances in paper partition chromatography and ion-exchange chromatography.

The eight chapter titles give an exhibit of the book's offering. After an Historical Introduction come: Chromatography of: Adsorption; Partition; Ion-exchange, and Miscellaneous Forms. The last three consider: Treatment of Colourless Substances; Process Development, and Chromatography in Industry. References at chapter ends are aided by a seventeen item bibliography on the book's last pages. The few tables are distributed through the text's body. There is a "token" index of less than two pages.

The author expects this newer version to be useful to both university students and post-graduate workers. It should be added that it can also well serve to brief those of more general interest in this new type of analysis.

B. CLIFFORD HENDRICKS

RISK AND GAMBLING: *The Study of Subjective Probability*, by John Cohen and Mark Hansel, *Members of the Psychology Faculty, University of Manchester, England*. Cloth. Pages x+153. 12.5×18.5 cm. 1956. Philosophical Library, Inc., 15 E. 40th Street, New York, N. Y. Price $3.50.

Somewhere in antiquity someone coined the phrase, "As dull as a statistics book." Probably the phrase is as nearly correct as any that has ever been made. *Risk and Gambling* however is a resounding exception. It is the type of book one sits and reads to the finish and wonders why someone has not been clever enough to write it before.

Despite certain implications in the title, the book is not a short course for "card sharps." Rather it is an experimental study of subjective probability and risk-taking. The first aim of the study is to discover, if possible, the way people actually choose, estimate, predict, judge or take risks when they are required to make judgments on partial information. The second aim is to trace the changes that occur in these activities as a person grows older. Finally, the authors hope to determine whether the "guesses" indicated above follow their own psychological rules and how these rules compare with the laws of mathematical probability.

In order to attain their aims and reach their conclusions the authors undertook and reported a number of experiments involving guessing such as the color of beads that might be drawn from a jar, which uncle probably sent the present, and the willingness of pedestrians to cross a street as cars approached at different distances. The relationship of age of subjects to the results of the experiments is then analyzed and discussed in light of certain historical facts about gambling and the behavior of gamblers.

Unfortunately the authors came to no definite conclusions. They do point out that there seems to be certain consistencies of behavior when the mind is faced with risk. But, they also point out that the consistencies will be identified only after further research. It reminds one of a dinner of steak topped off with pie and ice cream. Unfortunately the pie and ice cream were not served. The steak, however, was wonderful.

Every mathematician interested in statistics should read this book for a renewal of perspective.

GEORGE G. MALLINSON
Western Michigan College
Kalamazoo, Michigan

UNIVERSITY OF ILLINOIS STUDIES REFORMS IN MATHEMATICS TEACHING

Reforms in the curriculum and teaching of high school mathematics will be furthered by a grant of $277,000 to the University of Illinois announced today by Carnegie Corporation of New York.

The sum will be used by a University committee to continue its work of devising a new mathematics course for each of the four secondary years, preparing classroom text materials and teachers' manuals, and conducting training courses to acquaint teachers with the new teaching techniques to be developed.

The Illinois project, which has been under way for more than four years, has the goal of bringing the high school mathematics curriculum up to date by including topics from modern mathematics and by presenting the subject as an integrated body rather than as a group of isolated courses.

The project is directed by a committee of representatives of the colleges of education, engineering, and liberal arts and sciences at Illinois. Its staff has already experimented with revisions of the curriculum for high school freshman, sophomore, and junior classes. Up to the present all materials have been tested in five different high schools in Illinois and Missouri, with the cooperation of 14 teachers and almost 500 students.

John W. Gardner, president of Carnegie Corporation, hailed the work of the Illinois group as a "substantial attack on one of our most critical educational problems." The national shortage of scientists and engineers grows more acute each year, he pointed out, as fewer and fewer students enter college with adequate preparation in mathematics.

"The problem must be dealt with at its source," Gardner declared, citing recent studies, including one just published by the Educational Testing Service under a Carnegie grant, which describe most high school mathematics curricula and teaching as inadequate.

The Illinois grant is one of 10, amounting to almost three-quarters of a million dollars.

CORRESPONDENCE SCHOOLS

Twenty-five correspondence schools have just been accredited by the National Home Study Council, the accrediting association for private home study schools, it was announced by Dr. Homer Kempfer, Executive Secretary of the Council.

In recent months an Accrediting Commission of prominent educators thoroughly inspected all private home study schools which applied. Of the total group scattered throughout the nation, twenty-five schools were found to meet the rigid standards required for accreditation.

Nearly 700,000 people enroll in private correspondence schools annually, according to Dr. Kempfer. He pointed out that this is more than the number of freshmen enrolling in all colleges and universities of the country.

"With so many people enrolling, it is highly important for the public to know which schools are worthy of confidence," said Dr. Kempfer. "It is heartening to realize that over fifty percent of the enrollment is in accredited schools."

"If enrollment in correspondence schools continues to increase at the present rate, the number of students taking home study courses will double within the next ten years," declared Mr. Kempfer.

To be accredited by the National Home Study Council, a school must offer educationally sound and up-to-date courses, have a competent faculty, admit only qualified students, advertise truthfully, keep its tuition charges reasonable, show a good record of ethical relationships with students, and be financially sound.

The National Home Study Council has been the standard-setting agency for private home study schools for thirty years.

For further information about correspondence schools which meet high standards, people may inquire for the *Home Study Blue Book* at public libraries or of high school guidance counselors. A list of accredited schools is available without charge from the National Home Study Council, 1420 New York Ave., N.W., Washington 5, D. C.